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LABORATORY OF MEASUREMENTS

EXPERIMENT No **1**

Investigation of uncertainties in measurements of electrical quantities

GOAL

The goal of the experiment is to acquaint students with analysis of sources of errors and evaluation of the uncertainties in the direct and indirect measurement of electrical quantities, using two methods.

EQUIPMENT SPECIFICATION

Instrumentation and software

Instruments

1. Digital Multimeter type APPA 109N
2. Digital Multimeter type Metex M-3270D
3. Digital Multimeter with scanning and data logger function RIGOL DM3051 operated by computer via USB interface
4. Analogue Multimeter type: PROTEK 3030S
5. Power supply +/- 30V ; 5 A (voltage and current controlled)

Software:

1. RIGOL data acquisition software.
2. Microsoft Office Excel
3. LabVIEW - Laboratory Virtual Instrument Engineering Workbench

THEORETICAL BASIS

The objective of measurement is to assign one or more quantity values that can reasonably be attributed to a physical quantity in experimental way.

Due to imperfection of instruments used in measurement process, measurement methods, environmental impact and variability of environmental conditions, we can assign a certain interval, with a certain probability associated with the measurand.

The objective of measurement in the uncertainty approach is not to determine a true value as closely as possible. Rather, it is assumed that the information from measurement only permits assignment of an interval of reasonable values to the measurand, based on the assumption that no mistakes have been made in performing the measurement. Measurand is a quantity intended to be measured. The term "interval" is used together with the symbol [a; b] to denote the set of real numbers x for which a ≤ x≤ b, where a and b>a are real numbers. The term "interval" is used here for "closed interval". The symbols a and b denote the 'end-points' of the interval [a; b].

A measurement result is complete only when accompanied by a quantitative statement of its uncertainty; it should be expressed by two numbers. One – the best estimate of the value of quantity under measurement, while the second should characterize the dispersion which specifies the interval covering the value of quantity at a certain "*p*" coverage probability (level of confidence).

Correctly presented measurement result is as follows: (658,2±1,2) Ω *p* = 0,95. „658,2" is the best point estimator of the measured quantity, „±1,2" denotes the limits of the interval of estimator dispersion (Fig.1) at coverage level (level of confidence) of 0,95.

The interval can be also expressed as percentage of point estimator: (658,2 Ω ± 1,9 %) *p* = 0,95 or just limits in brackets: [657,0;694,4].

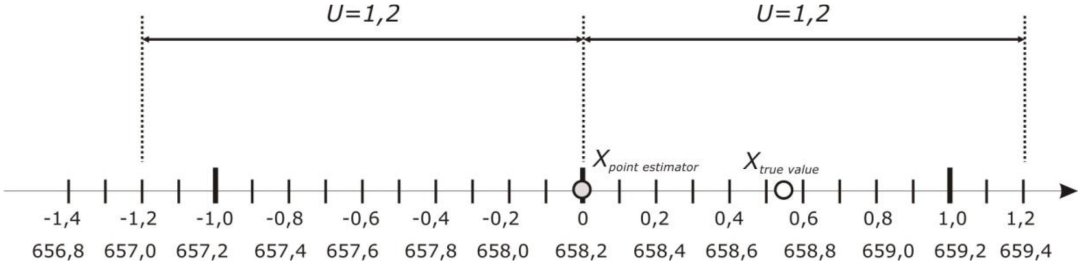


Fig.1. Graphical presentation of measurement result and uncertainty range. A point estimator it can be just a simple instrument reading or a mean value of series of observations

Rounding off the measurement result

Whenever a numerical value is quoted, the choice of number of digits is needed. Round-off the number by discarding some of the digits at its right-hand end is essential for measurement result presentation.

The rules for rounding off point estimator are as follows:

- If the first non-significant digit is less than 5, then the least significant digit remains unchanged
- If the first non-significant digit is greater than 5, the least significant digit is incremented by 1
- If the first non-significant digit is 5, the least significant digit can either be incremented or left unchanged
  - If there is any digit after 5 then the least significant digit is incremented by 1
  - Otherwise, is incremented by 1 if the least significant digit is odd and not incremented if the least significant digit is even

There are also rules for rounding-off uncertainty and general requirement is not to lower our confidence to the estimated interval, so round-up and keeping two significant digits is recommended.

<u>Example:</u>

Let's consider the case:

(62,831 853±0,104 719 76) mm at p=0,95

Firstly we round-off uncertainty - the rule is to round-up leaving two significant digits.

So, if from calculation, a raw value before rounding off is U=0,104 719 76

then after rounding up U = 0,11.

The uncertainty is determined with a certain level confidence. Rounding up of uncertainty is an effect of requirement of not lowering our confidence to measurement result.

Secondly we round off point estimator of the value of quantity:

the rule is to keep the same number of significant digits as in uncertainty, so

62,831 853 =≈ 62,83

The measurement result is quoted as: (62,83±0,11) mm at p=0,95

It is also permissible to keep one significant digit in uncertainty if the rounding up does not cause a rounding error to be bigger then 10% or rounded uncertainty.

Several examples of rounding off

Example 1

Raw from calculations	R= (107,5235 ± 0,00921) Ω
After rounding-off	R=(107,52 ± 0,01) Ω

Rounding up of uncertainty is below 10% (8,6%), so only one significant digit in uncertainty is presented.

Example 2

Raw from calculations	R=(107,5234 ± 0,015126) Ω
After rounding-off	R=(107,523 Ω ± 0,016) Ω

Rounding–up of uncertainty is over 10% (32%), so two significant digits are quoted in uncertainty.

Example 3

Raw from calculations	R=(107,52350001 ± 0,015126) Ω
After rounding-off	R=(107,524 ± 0,016) Ω

Example 4

Raw from calculations	R=(107,5225000 ± 0,015126) Ω
After rounding-off	R=(107,522 ± 0,016) Ω

Example 5

Raw from calculations	R=(107,5235000 ± 0,015126) Ω
After rounding-off	R=(107,524 ± 0,016) Ω

Example 6

Raw from calculations	R=(107,522501 ± 0,01500011) Ω
After rounding-off	R=(107,523 ± 0,016) Ω

Example 7

Raw from calculations	R=(107,52251 ± 0,015126) Ω
After rounding-off	R=(107,523 ± 0,016) Ω

Example 8

Raw from calculations	R = (376,35602 ± 0,12501) Ω
After rounding-off	R = (376,36 ± 0,13) Ω

Further examples of measurement data statements:

(127 ± 13) μm p=0,95

(23,2 ± 0,1) °C p=0,95

(230,4 ± 1,2) V p=0,95

(3,33 ± 0,12) s p=0,95

(50,46 ± 0,25) Hz p=0,95

p=0,95 is the most commonly used level of confidence (coverage factor).

Sources of errors and uncertainties:

- incomplete definition of the measurand
- imperfect realisation of definition of the measurand
- non-representative sampling – the sample measured may not represent the defined measurand
- personal bias in reading especially might appear if analogue instruments are used
- imperfection of human sensing abilities to read instrument indication and finite resolution of instruments,
- finite instrument resolution or discrimination threshold
- inexact values of measurement standard and reference material
- inexact value of constants and other parameters obtained from external sources and used in data-reduction algorithm
- approximations and assumptions incorporated in the measurement method and procedure (imperfection of measurement methods)
- incomplete knowledge about the impact of environment on the object of measurement and measuring instruments
- variation in repeated observations of measurand under apparently identical conditions
- imperfection of instruments used (data specification and certificate of instrument calibration help to evaluate uncertainty)
- human errors in instrument reading which might be classified as gross errors,

The boundaries defining the range of uncertainty, which are usually taken as symmetrical with respect to the point estimator of mark with an uppercase "*U*", and the confidence level lowercase "*p*".

General statement of measurement result: $x = \bar{x} \pm U$ is equivalent to $x \in \left\langle \bar{x} - U ; \bar{x} + U \right\rangle$.

An arithmetic mean of n observations of x_i of a quantity X is serving usually as point estimator – the best approximation of value of the quantity. The arithmetic mean is expressed by formula:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} .$$

Measurement uncertainty (uncertainty of measurement, or uncertainty) is a non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand, based on the information used. (definition given by International Vocabulary of Metrology — Basic and general concepts and associated terms, VIM 3rd edition, JCGM 200:2008)

NOTE 1 Measurement uncertainty includes components arising from systematic effects, such as components associated with corrections and the assigned quantity values of measurement standards, as well as the definitional uncertainty. Sometimes estimated systematic effects are not corrected for but, instead, associated measurement uncertainty components are incorporated.

NOTE 2 The parameter may be, for example, a standard deviation called standard measurement uncertainty (or a specified multiple of it), or the half-width of an interval, having a stated coverage probability.

NOTE 3 Measurement uncertainty comprises, in general, many components. Some of these may be determined by Type A evaluation from the statistical distribution of the quantity values from series of measurements, and can be characterized by standard deviations. The other components, which may be determined by Type B evaluation, can also be characterized by standard deviations, obtained from probability density functions, based on experience or other information.

NOTE 4 In general, for a given set of information, it is understood that the measurement uncertainty is associated with a stated quantity value attributed to the measurand. A modification of this value results in a modification of the associated uncertainty.

Uncertainty U is called the expanded measurement uncertainty, and the adjective "expanded" comes from a fairly popular method of its determination according to the formula

$$U = k_p u_c ,$$

where k_p is a coverage factor, which is a number larger than one by which a combined standard measurement uncertainty u_c is multiplied to obtain an expanded measurement uncertainty. Coverage factor can be calculated for level of confidence "*p*", called coverage probability.

u_c is a combined standard measurement uncertainty that is obtained using the individual standard measurement uncertainties associated with the input quantities in a measurement model

u_c is a combined uncertainty as:

$$u_c^2 = u_A^2 + u_B^2 .$$

u_A – Type A evaluation - often called Type A standard uncertainty, and is calculated from series of repeated observations and is the familiar statistically estimated variance s^2

$$s = s_{n(n-1)} = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n(n-1)}}$$

u_B – Type B evaluation - often called a Type B standard uncertainty.

<u>Uncertainty calculated by Type A method and Type B method.</u>

Evaluation of type A uncertainty is based on series of observations. u_A is an estimated standard deviation, which is the positive square root of variance (Var=s^2=u_A^2) of observations thus $u_A = s$ and for convenience is sometimes called a *Type A standard uncertainty*.

Type B standard uncertainty is the pool of information, which may include: previous measurement data; experience with or general knowledge of the behaviour and properties of relevant materials and instruments; manufacturer's specifications; data provided in calibration and other certificates; uncertainties assigned to reference data taken from handbooks.

<u>Uncertainties of type A</u>

Calculation of type A uncertainties requires a set of values coming from experimental observations of the quantity. Measurements must be repeated in the same conditions. The observations, which are the readings from the measuring instrument, carried out in the same environmental conditions, should differ in some way in values, and differences between them should have a random character. The observations should not carry any a'propri (known in advance) systematic influences - trends or correlation effect. A set of data can be treated as random variable and statistical methods are applied to evaluate uncertainty of type A.

If X is a set of observations: x_1, x_2, x_3, … x_n then the arithmetic mean of n individual readings can be regarded a the best estimate of a quantity value under measurement.

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \text{ - arithmetic mean of } n \text{ observations} \qquad (1)$$

The standard deviation s_{n-1} of a measurement sample is a subset of the population containing all possible values of observation. For continuous random variable infinitely large number is possible, but it would require infinitely long period to carry out the observations.

The deviation of the single observation in the sample is expressed by:

$$s_{n-1} = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n-1}} \text{ where } \bar{x} \text{ - is an arithmetic mean of } n \text{ observations} \qquad (2)$$

Standard uncertainty u_A equals to standard deviation of mean from n- observations and is express by formula:

$$u_A = s_{n(n-1)} = \frac{s_n}{\sqrt{n}} = \frac{1}{\sqrt{n}}\sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n\left(n-1\right)}} \qquad (3)$$

When infinitely large number of influences which affect individual observations, and encountered during the measurement process, and the number of observations is equally large, the dispersion of results corresponds to Gauss distribution. Otherwise the t-Student distribution should be applied.

In practice in many applications the t-Student distribution is applied when the number of observations n<30.

Type A standard uncertainty is equal to the standard deviation of average of n observations $u_A = s_{n(n-1)}$

For Gauss distribution, the coverage factor, k_p for a coverage probability $p = 0,95$ is: k_p =1,96.

Cumulative distribution of Gauss probability density function is given by (4),

$$P(x)=\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\left(\frac{x^2}{2\sigma^2}\right)}dx \qquad (4)$$

Where: σ is a standard deviation

Probability density function of (4) is given by (5):

$$p(x)=\frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{x^2}{2\sigma^2}\right)} \qquad (5)$$

Graphically it is presented in Fig. 2a and Fig. 2b.

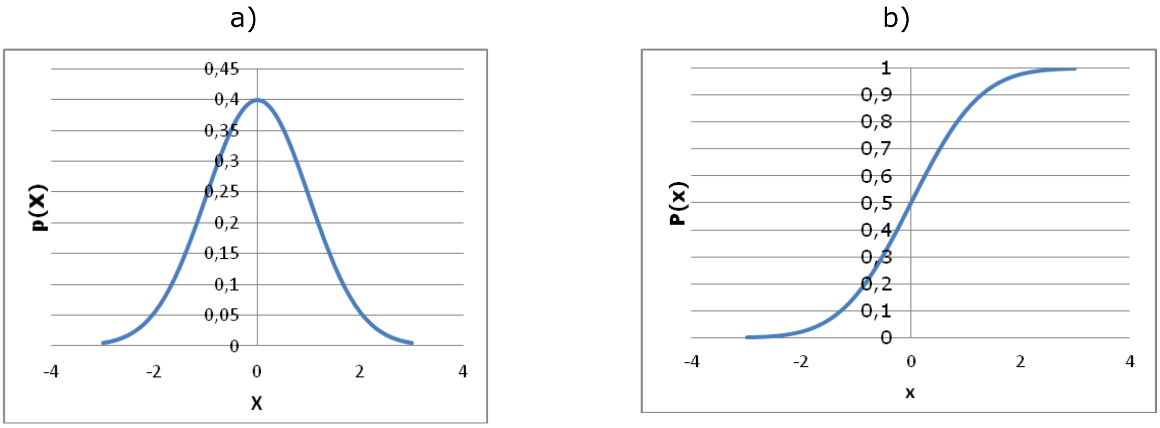


Fig. 2. Standardized Gaussian distribution $N(0,1)$ it means Gauss for $\bar{x}=0$ and $\sigma=1$: (a) probability density function $p(x)$, (b)cumulative probability density function, $P(x)$.

Probability density function (PDF) for normalized Gauss PDF is called Normal distribution and is denote as capital „*N*" with parameters in brackets $N(0,1)$: where mean value $\bar{x}=0$, standard deviation $\sigma=1$.

For $N(0,1)$ coverage factors k_p for a several levels of confidence p are in Tab. 1.

Tab. 1. Coverage factors k_p for a several levels of confidence p.

p	0,5	0,683	0,95	0,99	0,997	0,999
k_p	0,676	1	1,96	2,58	2,97	3,29

As Gaussian distribution is used for n >30, than in tab. 1a. $k_{p,(n-1)}=k_{0,95;\nu}$ for coverage probability of $p=0{,}95$ (most common used) for a chosen number of degrees of freedom, ν, which equals $\nu=n-1$ where n – number of observations are given in tab. 1a

Tab. 2. Coverage factors $k_{p,\nu}$ for *t*-Student distribution ($\nu = n$-1 where *n* – number of observations).

ν	$p=0.90$	$p=0.95$	$p=0.99$	$p=0.999$
1	6.31	12.71	63.66	636.62
2	2.92	4.30	9.93	31.60
3	2.35	3.18	5.84	12.92
4	2.13	2.78	4.60	8.61
5	2.02	2.57	4.03	6.87
6	1.94	2.45	3.71	5.96
7	1.89	2.37	3.50	5.41
8	1.86	2.31	3.36	5.04
9	1.83	2.26	3.25	4.78
10	1.81	2.23	3.17	4.59
11	1.80	2.20	3.11	4.44
12	1.78	2.18	3.06	4.32
13	1.77	2.16	3.01	4.22
14	1.76	2.14	2.98	4.14
15	1.75	2.13	2.95	4.07
16	1.75	2.12	2.92	4.02
17	1.74	2.11	2.90	3.97
18	1.73	2.10	2.88	3.92
19	1.73	2.09	2.86	3.88
20	1.72	2.09	2.85	3.85
21	1.72	2.08	2.83	3.82
22	1.72	2.07	2.82	3.79
23	1.71	2.07	2.82	3.77
24	1.71	2.06	2.80	3.75
25	1.71	2.06	2.79	3.73
26	1.71	2.06	2.78	3.71
27	1.70	2.05	2.77	3.69
28	1.70	2.05	2.76	3.67
29	1.70	2.05	2.76	3.66
30	1.70	2.04	2.75	3.65
40	1.68	2.02	2.70	3.55
60	1.67	2.00	2.66	3.46
120	1.66	1.98	2.62	3.37
∞	1.65	1.96	2.58	3.29

ν - is a degree of freedom and is equal to number of observations minus 1

$\nu = n$-1

Comparison of PDFs of Gauss *N* (0,1) distribution and *t*-Student distributions

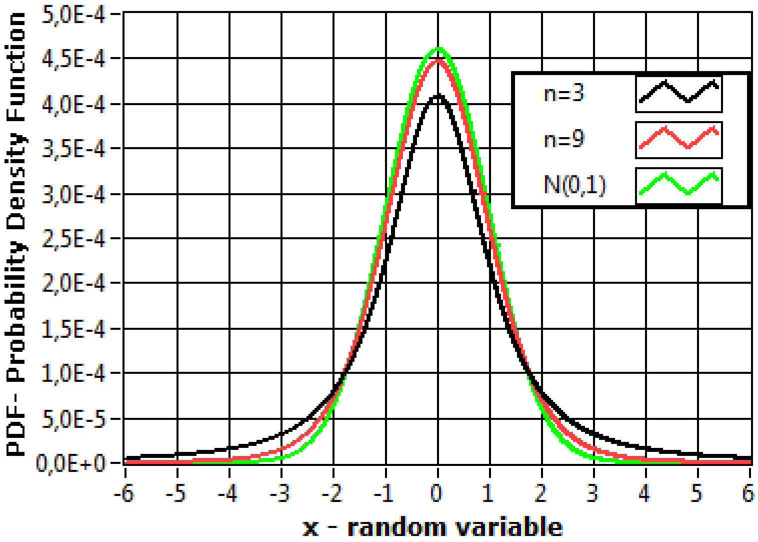
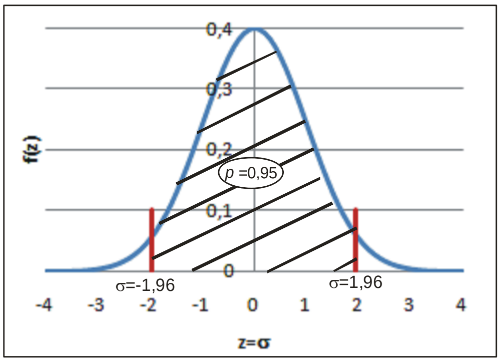


Fig. 3. Probability density function for: *t*-Student distribution for $s_{n(n-1)}=1$ for *n*=3 (ν =2) and *n*=9 (ν =8) observations and Normal distribution *N*(0,1).



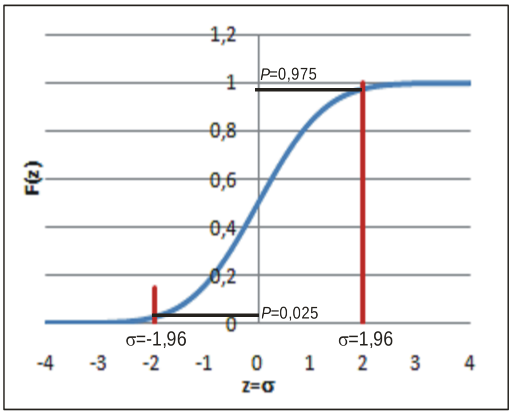


Fig 3a. Probability density function of continuous random variable Function of Normal distribution N(0,1) a) probability density function, *f*(z) b) cumulative probability density function *F*(*z*)

<u>**Uncertainties of type B**</u>

The type B method of uncertainty evaluation, requires the information of the measurement errors coming from other sources than a series of observations. Sources of such information can be obtained from:

- previous measurement data;
- experience with or general knowledge of the behaviour and properties of relevant materials and instruments;

- manufacturer's specifications;
- data provided in calibration and other certificates;
- uncertainties assigned to reference data taken from handbooks.

There are at least one or two parameters specified in instrument technical data sheet supplied together with every instrument , which allows to calculate maximum permissible measurement error, also called limit of error. Basing on these parameters it is possible to calculate limits of errors for each instrument reading. These limits, also called maximum permissible errors, usually are symmetrically located around the indicated value by instrument.

The maximum permissible measurement error is expressed by following formula:

$$\Delta_{\max P} = \frac{1}{100}\delta_m x_{rdg} + \frac{1}{100}\delta_a x_{FSR}$$

where:

δ_m , is a multiplicative component (expressed in %)

δ_a - is a additive component (expressed in %)

x_{rdg} - is a instrument reading (value indicated by instrument) in measuring unit

x_{FSR} - is a full scale range (in measuring units)

$\Delta_{\max P}$ - is a maximum permissible measurement error expressed in measuring unit

The maximum permissible error allows to determine borders of the interval, which covers the measured value. The borders are located symmetrically around the value indicated by the instrument therefore the interval is: $\left\langle x_{rdg} - \Delta_{\max P}; x_{rdg} + \Delta_{\max P} \right\rangle$. The question is: "what is the probability that the interval covers the measured value?". According to instrument technical specification no outside of specified coverage interval is expected, so coverage probability equals to 1 (p=1). The errors inside coverage interval are uniformly distributed as given in Fig. 4a. The relation presented in Fig 4a is called probability density function and the function in Fig. 4b which is an integral of relation from Fig 4a is called a cumulative probability density function or distribution function.

If probability density function is a straight horizontal line over interval $\left\langle x_{rdg} - \Delta_{\max P}; x_{rdg} + \Delta_{\max P} \right\rangle$, the cumulative probability density function is a skew line starting at low boundary of coverage interval with $P(x)$=0 ending at higher boundary of coverage interval at $P(x)$=1.

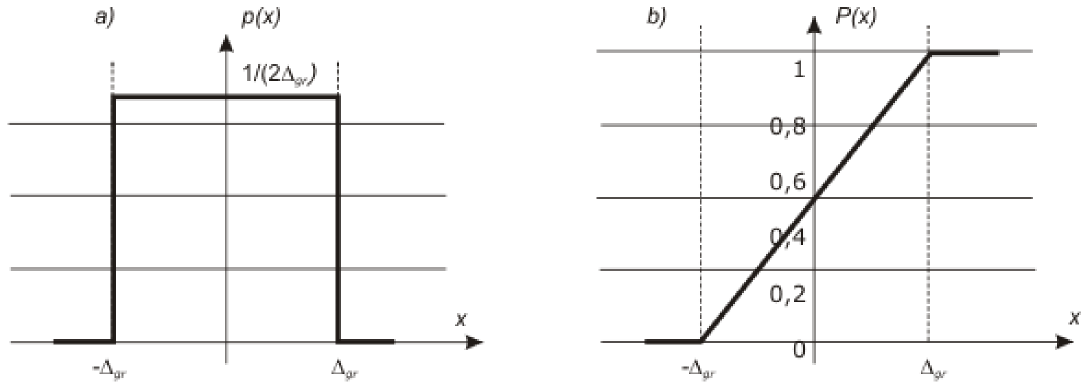


Fig. 4. a) probability density function *p(x)* of uniform distribution b) cumulative distribution function $P(x)$of uniform distribution

Standard deviation σ is a square root of variance v. Definition of variance is expressed as follows:

$$v = \sigma^2 = \int_{-\infty}^{\infty} p(x)\left(x - \bar{x}\right)^2 dx \text{, so } \sigma = \sqrt{v} = u \text{ .}$$

Using the above according to the Fig. 4.a: $p(x)=1/(2\Delta)$ the standard uncertainty of type B, u_B, of uniformly distributed $p(x)$ is calculated as follows:

$$u_B^2 = \sigma^2 = \int_{x_{rdg}-\Delta_{\max P}}^{x_{rdg}+\Delta_{\max P}} \frac{1}{2\Delta_{\max P}} x^2 dx = \frac{1}{2\Delta_{gr}} \left.\frac{x^3}{3}\right|_{x_{rdg}-\Delta_{\max P}}^{x_{rdg}+\Delta_{\max P}} = \frac{1}{2\Delta_{\max P}} \frac{\left(\Delta_{\max P}\right)^3 - \left(-\Delta_{\max P}\right)^3}{3} = \frac{\Delta^2}{3} \text{ or}$$

$$u_B = \sqrt{u_B^2} = \frac{\Delta_{\max P}}{\sqrt{3}} \tag{6}$$

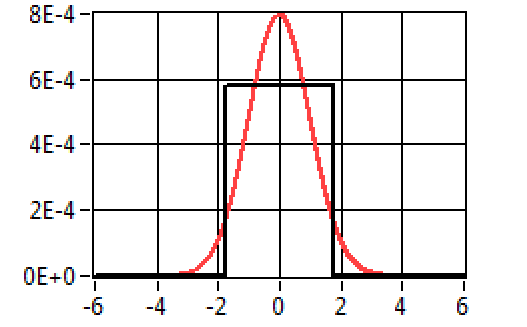
Combined standard uncertainty

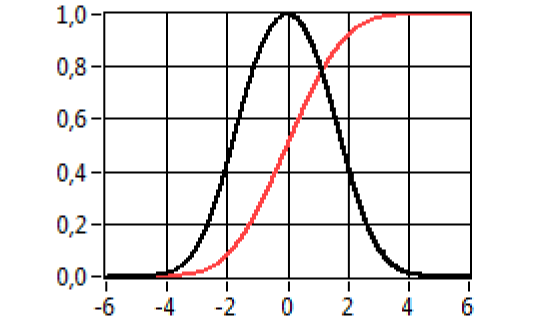
Combined standard uncertainty is calculated as geometrical sum of two components (7):

$$u_c = \sqrt{u_A^2 + u_B^2} \tag{7}$$

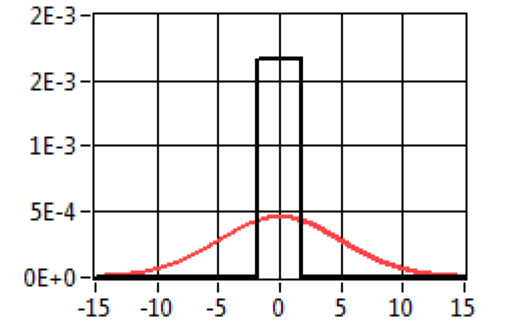
If both components u_A and u_B in (7) are coming from two different not Gaussian type distributions, the u_c. belongs to third type of distribution. If u_A and u_B are two Gaussian distributions with different values of standard deviation, then u_c also is a Gaussian type distribution.

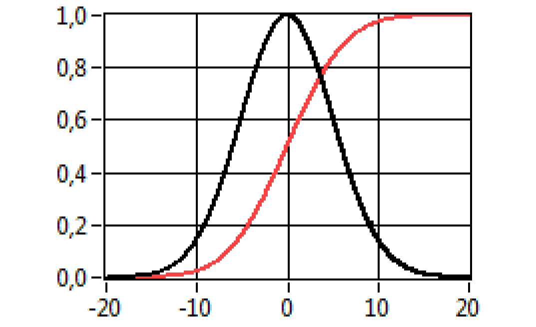
In left column are different probability density functions. In right columns are probability distribution functions referred to combined probability distribution function components of which are A and B (in black and in red cumulative probability function of combined PDF.



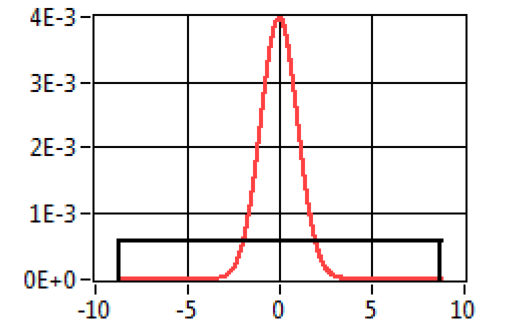


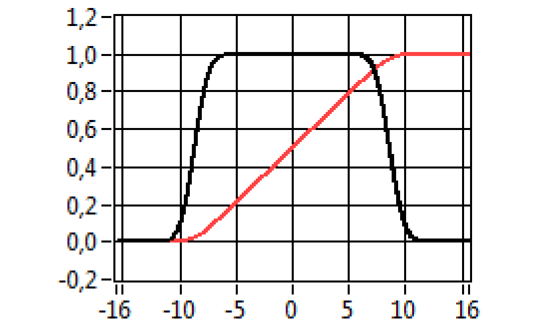
Uniform PDF of u_B is black; Gauss PDF *(u_A) is red; ratio:* u_A/u_B =1/1	$k_{p=0,95}$=1,92 for p=0,95 $U_{p=0,95} = k_{p=0,95} \cdot u_c = k_{p=0,95}\sqrt{u_A^2 + u_B^2}$



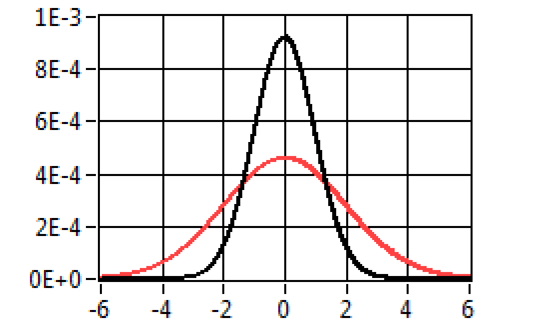


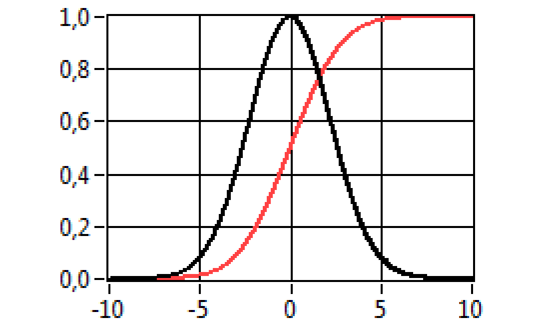
Uniform PDF of u_B is black; Gauss PDF *(u_A) is red; ratio:* u_A/u_B =1/2	$k_{p=0,95}$=1,96 for p=0,95 $U_{p=0,95} = k_{p=0,95} \cdot u_c = k_{p=0,95}\sqrt{u_A^2 + u_B^2}$



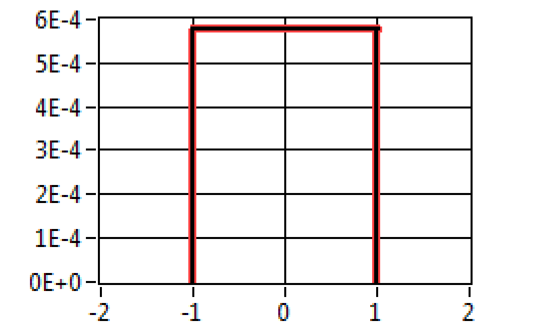


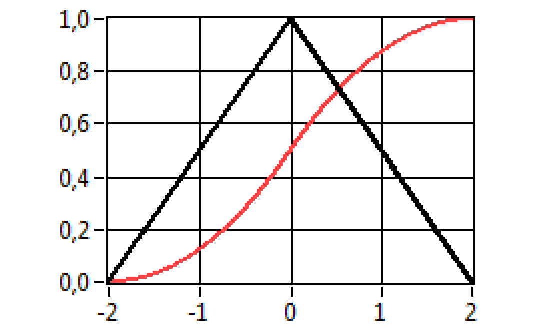
Uniform PDF of u_B is black; Gauss PDF *(u_A) is red; ratio:* u_A/u_B =1/5	k_p=1,69 for p=0,95 $U_{p=0,95} = k_{p=0,95} \cdot u_c = k_{p=0,95}\sqrt{u_A^2 + u_B^2}$



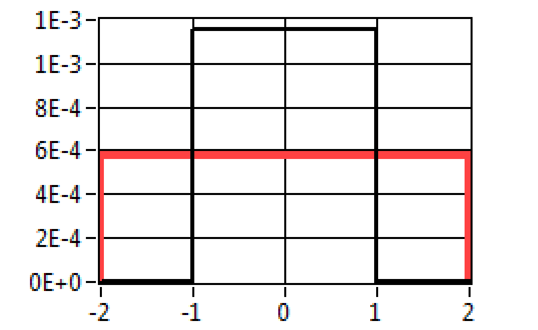


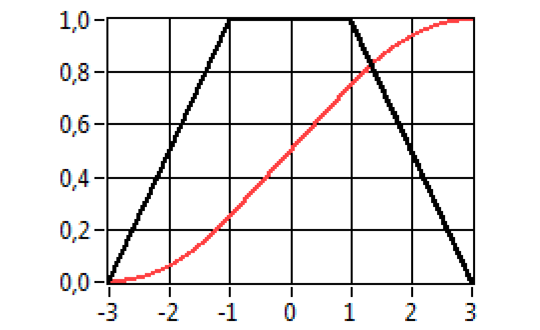
$u_{A1}/(u_{A2})=1/2$ both distributions are Gaussian type	$k_p=1{,}96$ for p=0,95 $U_{p=0,95}=k_{p=0,95}\cdot u_c=k_{p=0,95}\sqrt{u_{A1}^2+u_{A2}^2}$





$u_{B1}/u_{B2}=1/1$ (but both distributions are uniform over interval <-1;1>	$k_p=1{,}90$ for $p=0{,}95$ $U_{p=0,95}=k_{p=0,95}\cdot u_c=k_{p=0,95}\sqrt{u_{B1}^2+u_{B2}^2}$





$u_{B1}/u_{B2}=1/2$ (but both distributions are uniform over interval <-1;1>	$k_{p=0,95}=1{,}84$ for $p=0{,}95$ $U_{p=0,95}=k_{p=0,95}\cdot u_c=k_{p=0,95}\sqrt{u_{B1}^2+u_{B2}^2}$

The coverage interval in most measurements is evaluated for the coverage probability 0,95. It means our confidence is 95 % that measured quantity value is covered by the interval.

Standard measurement deviation of indirect measurements

Indirect measurements are the measurements in which is determined both from measurement values and mathematical operations.

For example: R=V/I is an indirect measurement of R when V and I are read.

Indirect measurements like: $R=V/I$, to calculate standard deviation it is necessary to use the following relation for composite function.
Consider the general relation, that $y=f(x_1, x_2, x_3, x_4 \ldots x_n)$

That standard deviation for not correlated quantities X_1, X_2, X_3, $X_4 \ldots X_n$ is given by

$$u^2(y) = \left(\frac{\partial f(x_1, x_2, x_3, \ldots x_n)}{\partial x_1}\right)^2 u_{x_1}^2 + \left(\frac{\partial f(x_1, x_2, x_3, \ldots x_n)}{\partial x_2}\right)^2 u_{x_2}^2 + \left(\frac{\partial f(x_1, x_2, x_3, \ldots x_n)}{\partial x_3}\right)^2 u_{x_3}^2 + \ldots.. \left(\frac{\partial f(x_1, x_2, x_3, \ldots x_n)}{\partial x_n}\right)^2 u_{x_n}^2 \quad (8)$$

or in the form:

$$u^2(y) = c_{x_1}^2 u_{x_1}^2 + c_{x_2}^2 u_{x_2}^2 + c_{x_3}^2 u_{x_3}^2 + \ldots + c_{x_n}^2 u_{x_n}^2 \quad (9)$$

Where ; $c_{x_i} = \frac{\partial f(x_1, x_2, x_3, \ldots x_n)}{\partial x_i}$ - called sensitivity coefficients

Practical remark: If distribution is not far removed from Gaussian shape a $k_p=2$ for $p=0{,}95$ can be used, thus $U=2u_c$.

As an example let us calculate uncertainty for measurement of resistance applying indirect method:

$$R = \frac{V}{I}$$

Resistance depends on two input quantities: V and I, thus R=f(V,I), so if only single readings (not a series of observations) are available from instruments, and if V and I are not correlated, only u_B: can be calculated from:

$$u_{Bc}^2(R) = \left(\frac{\partial f(R))}{\partial V}\right)^2 u_{BV}^2 + \left(\frac{\partial f(R)}{\partial I}\right)^2 u_{BI}^2 \quad (10)$$

Or with sensitivity coefficients:

$$u_{Bc}^2(R) = (c_V)^2 u_{BV}^2 + (c_I)^2 u_{BI}^2 \quad (11)$$

c_V and c_I coefficients can be calculated from (12)

$$c_V = \frac{\partial f(R)}{\partial V} = \frac{1}{I} \qquad c_I = \frac{\partial f(R)}{\partial I} = V\left(-\frac{1}{I^2}\right) = \frac{-V}{I^2} \quad (12)$$

Inserting (12) to (11) we obtain (13):

$$u_{Bc}^2(R) = \left(\frac{1}{I}\right)^2 u_{Bv}^2 + \left(\frac{-V}{I^2}\right)^2 u_{BI}^2 \quad (13)$$

It is more convenient for further calculations of (13) if left and right sides of (13) are divided by $R=V/I$ (14). „rel" refers to relative value.

$$u_{Bc\,rel}^2(R)=\frac{u_{Bc}^2(R)}{R^2}=\frac{\left(\frac{1}{I}\right)^2 u_{Bv}^2}{R^2}+\frac{\left(\frac{-V}{I^2}\right)^2 u_{BI}^2}{R^2}$$ and next

$$u_{Bc\,rel}^2(R)=\frac{u_{Bc}^2(R)}{R^2}=\frac{\left(\frac{1}{I}\right)^2 u_{Bv}^2}{\left(\frac{V}{I}\right)^2}+\frac{\left(\frac{-V}{I^2}\right)^2 u_{BI}^2}{\left(\frac{V}{I}\right)^2}=\frac{u_{Bv}^2}{V^2}+\frac{u_{BI}^2}{I^2}=u_{BV\,rel}^2+u_{BIrel}^2 \qquad (14)$$

So (14) can be presented as:

$$u_{Bc\,rel}(R)=\sqrt{u_{BV\,rel}^2+u_{BIrel}^2} \qquad (15)$$

And:

$$u_{BV\,rel}=\frac{\Delta_{\max P}V}{\sqrt{3}\cdot V_{rdg}} \qquad u_{BIrel}=\frac{\Delta_{\max P}I}{\sqrt{3}\cdot I_{rdg}} \qquad (16)$$

And finally B type standard measurement uncertainty in ohms:

$$u_{Bc}=u_{Bc\,rel}\cdot R \qquad (17)$$

Coverage factor k_p depends on combined distributions of components. Both are uniformly distributed and the combination is a trapeze shape.

Tab. 3 Coverage factors, $k_{p=0,95}$, for two uniform distributions several ratios of u_{B1}/u_{B2}.for coverage probability of p=0,95

u_{B1}/u_{B2}	0:1	0,1:1	0,2:1	0,3:1	0,4:1	0,5:1	0,6:1	0,7:1	0,8:1	0,9:1
$k_{p=0,95}$	1,645	1,652	1,698	1,751	1,796	1,834	1,862	1,881	1,894	1,900

u_{B1}/u_{B2}	1:1	1:2	1:3	1:4	1:5	1:6	1:7	1:8	1:9	1:10
$k_{p=0,95}$	1,902	1,834	1,767	1,724	1,698	1,662	1,670	1,662	1,656	1,652

For a uniform distribution (sometimes called rectangular distribution) the borders of coverage interval: $<x_{rdg}-U_p;\ x_{rdg}+U_p>$ for p coverage probability, U_p is expressed: $U_p = k_p u_c$ where $k_p = p\sqrt{3}$ for $p \in <0;1>$

For ex ample: $k_{0,95}=0{,}95\sqrt{3}$

In the case of two uniform distributions of which combined standard uncertainty $u_{cB}=\sqrt{u_{B1}^2+u_{B2}^2}$ or $u_{cBrel}=\sqrt{u_{B1rel}^2+u_{B2rel}^2}$ the k_p factors are given in Tab. 3.

For combined uncertainty (relative of absolute values) of which components are as in (19)

$$u_c = \sqrt{u_A^2 + u_B^2} \tag{18}$$

Expanded uncertainty U at level for coverage probability p depends on standard uncertainty u (for combined $u = u_c$) and coverage factor k (k_p)

$$U_p = k_p u_c \tag{19}$$

k_p depends on combined distributions of u_c of components of type A and B.

If type A distribution (Gaussian or t-Student) are dominant then the coefficients from Tab 1 or Tab 2, can be used.

If uniform distribution is dominant then $k_p = p\sqrt{3}$.

If B type distribution is composed of two uniform distributions which are dominating over type A, then $k_{p=0,95}$ are given in tab. 3. Otherwise the approximate value for p=0,95 can be applied: $k_{p=0,95}=2$.

EXPERIMENT:

The measurement of an unknown value of resistor applying indirect and direct measurement methods, and estimation of uncertainties according to international guide: GUM - Guide to the expression of uncertainty in measurement *(GUM) JCGM OIML" 1993,* is a goal of experimental tasks

TASK 1:

Direct method measurement- two wire method

Resistance is to be measured by 4 instruments, of which maximum permissible errors $\Delta_{(\max P)R}$ are given by individual formulas as follows:

1) PROTRK type 3030S – analogue multimeter: $\Delta_{(\max P)R} = \pm 3\% R_{rdg}$
2) METEX type M3270D $\Delta_{(\max P)R} = \pm\left(0{,}8\% R_{rdg} + 2c_n\right)$
3) APPA type 109N – $\Delta_{(\max P)R} = \pm\left(0{,}3\% R_{rdg} + 3c_n\right)$
4) Rigol DM 3051 ($5\frac{3}{4}$ digit) – $\Delta_{(\max P)R} = \pm\left(0{,}015\% R_{rdg} + 0{,}006\% R_n\right)$ apply range

   R_n=4,00000 kΩ

   x_{rdg} - instruments reading;

   x_n - nominal range of instrument;

   c_n - least significant digit of instrument

Tab. Measured values.

TYPE of instrument	Instrument reading	Max. permissible instrument error	Instrument standard uncertainty	Relative instrument standard uncertainty	Expanded uncertainty at level of confidence p=0,95	Measurement result for p=0,95 $R = R_{rdg} \pm U$
	R_{rdg}	$\Delta_{(\max P)R}$	$u_B = \frac{\Delta_{(\max P)R}}{\sqrt{3}}$	$u_{B\,rel} = \frac{u_B}{R_{rdg}}$	$U = 0{,}95\sqrt{3}u_B$	
	Ω	Ω	Ω	%	Ω	Ω
PROTEK HC-3030S						
METEX M-3270D						
APPA 109N						
Rigol DM3051						

TASK 2

Indirect method measurement- four wire method

Measure resistance of the same resistor as in task 1, but applying indirect measurement method, using the same instruments as in Task 1, but using voltmeter and ammeter ranges of combining instruments in pairs as follows:

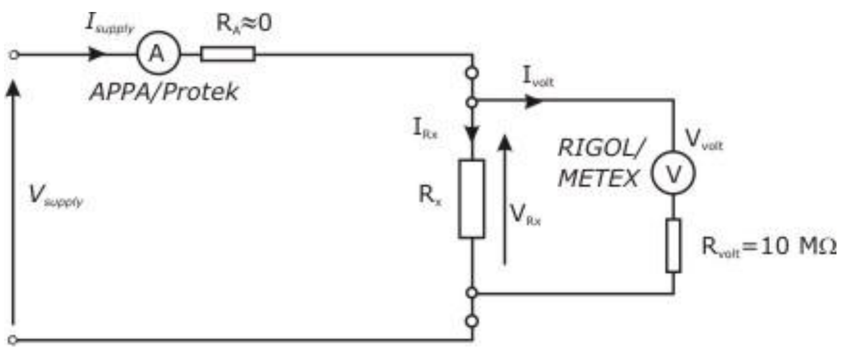
(i) RIGOL (as voltmeter) and APPA (as ammeter)

(ii) RIGOL (as voltmeter) and Protek (as ammeter)

(iii)METEX (as voltmeter) and APPA (as ammeter)

(iv) METEX (as voltmeter) and Protek (as ammeter)

Apply wiring as in Fig. 5



Fir. 5. Wiring for indirect resistance, R_x, measurement method R_x – resistor under test; I_{RX} – current passing through R_x, V_{Rx} voltage across R_x, A- ammeter, V- voltmeter, $V_{zaś}$=5V voltage of power supply $I_{zaś}$ current of power supply, V_{wolt} – voltage across voltmeter R_{wolt} – internal resistance of voltmeter; I_{wolt} – current through voltmeter

APPA and PROTEK multimetrs serving as ammeters

RIGOL i METEX multimetrs serving as voltmeters.

Tab. 2. RIGOL R_{VRIGOL} = 10 MΩ   METEX R_{VMETEX} = 10 MΩ

Voltmeters	V_V Voltage	$u_{V\,rel}$	***Ammeters***	I_A	$u_{I\,rel}$	$u_{R\,rel}$	$U_{Rx\,rel}$	U_{Rx}
	V	%		mA	%	%	%	Ω
RIGOL			APPA					
RIGOL			Protek					
METEX			APPA					
METEX			Protek					

Instrument's data referring to measurement instrument accuracy for selected ranges

Instrument type	Measurement range	$\Delta_{\max P}$ - maximum permissible instrument error	Borders of maximum, permissible instrument errors for voltage and current selected ranges of multimeters
RIGOL 3051	40 V	$\Delta_{\max P} = \pm\left(\delta_m x_{rdg} + \delta_a x_n\right)$	$\Delta_{(\max P)V} = \pm\left(0{,}0025\% x_{rdg} + 0{,}006\% x_n\right)$
METEX M-327D	40 V	$\Delta_{\max P} = \pm\left(\delta_m x_{rdg} + l_{ca} c_n\right)$	$\Delta_{(\max P)V} = \pm\left(0{,}8\% x_{rdg} + 2c_n\right)$
APPA 109	20 mA	$\Delta_{\max P} = \pm\left(\delta_m x_{rdg} + l_{ca} c_n\right)$	$\Delta_{(\max P)I} = \pm\left(0{,}20\% x_{rdg} + 40c_n\right)$
Protek 3030S	30 mA	$\Delta_{\max P} = \pm\left(\delta_a x_n\right)$	$\Delta_{(\max P)I} = \pm 3\% x_n$

Tab. 2a table of calculated resistances and uncertinties:

Instruments	$R = \frac{V}{I}$	$u_{BR\,rel} = \sqrt{u^2_{BV\,rel} + u^2_{BI\,rel}}$	$u_{BRx} = u_{BR\,rel} R$	$U_{B\,Rx} = k u_{BRx}$	Measurement statement $R_x \pm U_{B\,Rx}$
	Ω	%	Ω	Ω	Ω
RIGOL + APPA					
RIGOL + Protek					
METEX + APPA					
METEX + Protek					

x_{rdg} - instrument reading; x_n - instrument range/ sub-range

l_n - number of least significant digits; c_n - the least significant unit instrument sub- range δ_m - multiplicative component of instrument maximum permissible error; δ_a - additive component of instrument maximum permissible error

$$u_{V\,rel} = \frac{1}{\sqrt{3}} \cdot \frac{\Delta_{\lim\,voltmeter}}{U_{rdg\,voltmeter}} \qquad u_{I\,rel} = \frac{1}{\sqrt{3}} \cdot \frac{\Delta_{\lim\,ammetr}}{I_{rdg\,ammetr}}$$

$$U_{Rx\,rel} = k \cdot u_{Rx\,rel} \qquad u_{Rx\,rel} = \sqrt{u^2_{V\,rel} + u^2_{I\,rel}}$$

$$U_{Rx} = U_{Rx\,rel} \cdot R_x$$

Note: For rectangular error distribution at level of confidence *p*=0,95 coverage factor *k*=0,95 √3. If one of rectangular distributions is dominating *k*=0,95 √3 can by applied otherwise *k*=2

Consider the influence of power consumption by voltmeter circuitry and compare to current through resistor under test.

Current through voltmeter can be calculated from formula:

$$I_V = \frac{U_V}{R_V} =$$

Current leakage through voltmeter effect as indirect method measurement error, have a systematic nature, its value is known and measurement result can be corrected by that value.

Task 3

<u>Direct method measurement- four wire method</u>

Measurement wiring of RIGOL DM3051 is presented in Fig. 6. Record 100 observations in time intervals of 1 s; apply ULTRALOGGER data acquisition Rigol software to collect and record data in PC computer connected to instrument via USB interface.

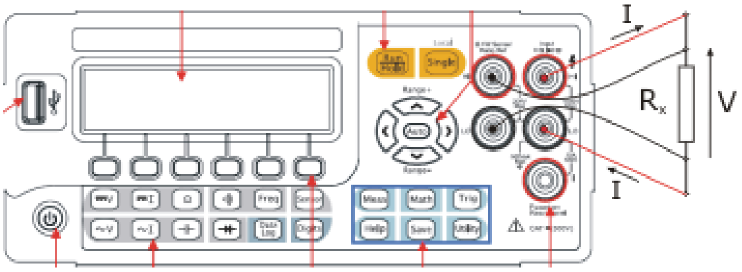


Fig 6. Four terminal direct resistance measurement method.

For data handling use and EXCEL package. Fit to Excel recoded txt of 100 observations (data) collected from RIGOL.

1. Draw "raw data" (observations) as a function of time, $R_i = f(t_i)$; t_i time of each collected observation (Fig. 7)
2. Check if any trends are observed (rys. 7)
   a. If any trend resent, remove the trend as it is a systematic effect, like a too short time of instrument heating after switching to power supply. Go to point 3
   b. If no trend is observed go to point 3

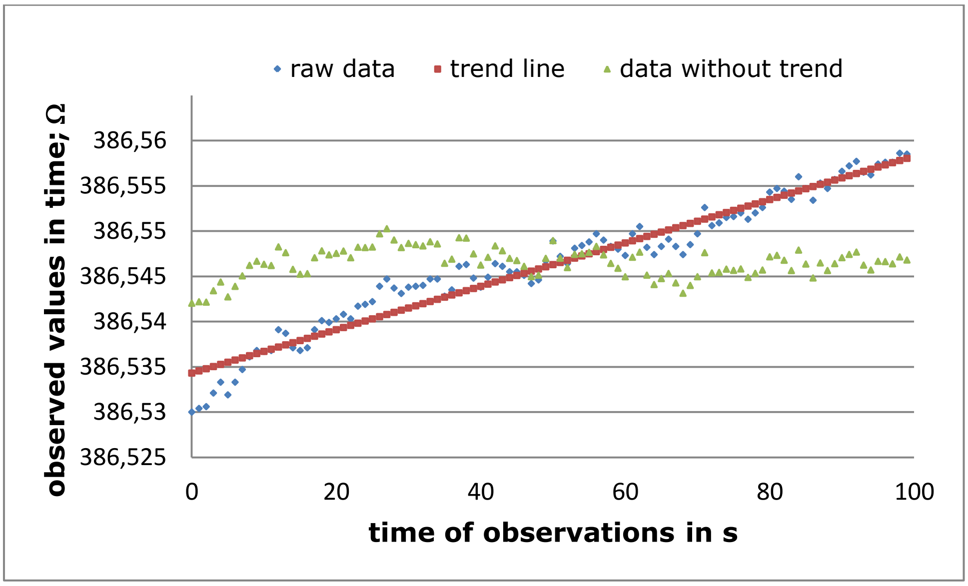


Fig 7. Raw data in time, identified trend and data without trend – corrected data

3. Calculation of mean value of corrected data: $\overline{R} = \frac{\sum_{i=1}^{n} R_i}{n}$;

4. Apparent errors as a difference between each observed value and mean, from point 3: $\Delta_i = R_i - \overline{R_x}$

5. Graph of apparent errors as a function in time $\Delta_i = f(t_i)$

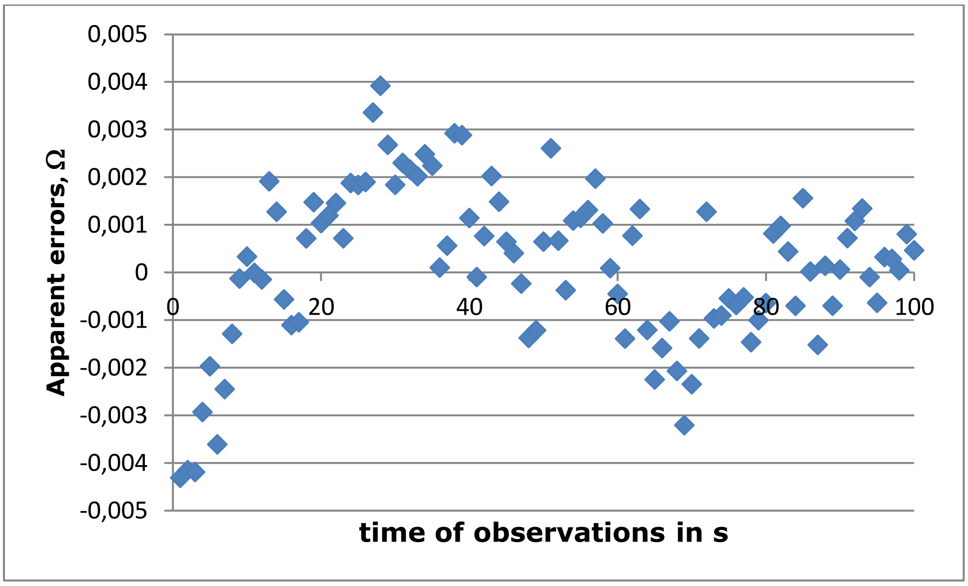


Fig 7a. Apparent errors in time

6. Standard deviation of apparent errors: $s_{(n-1)} = \sqrt{\dfrac{\sum_{i=1}^{n}(\Delta_i)^2}{n-1}}$

7. Check if in a set of apparent errors, Δ_i for $i = 1,\ 2,\ 3 \ldots n$ are any gross errors? Gross error might be recognized as errors which do not confirm relation: $|\Delta_i| < 3s_{(n-1)}$ for $i = 1,\ 2,\ 3 \ldots n$
   a. If any gross error is identified, eliminate from the raw observation these values from the set, and start the procedure from point 1
   b. Otherwise go to point 8.

8. Calculation of frequency of occurrences of apparent errors around mean value applying 11 intervals covering the whole range of apparent errors. Use Histogram function from Excel. Histogram present in graphical form (Fig. 8)

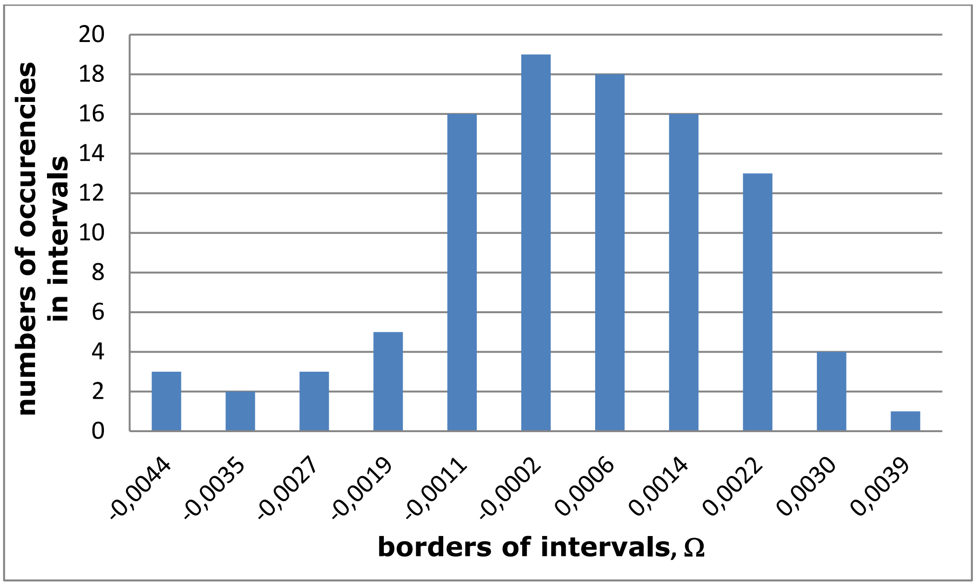


Fig 8. Numbers of occurrences of apparent errors in intervals

9. Based on histogram draw a frequency of occurrences of apparent errors in intervals, in relation to all observations. (division by 100 if no gross errors observed), (rys.9).

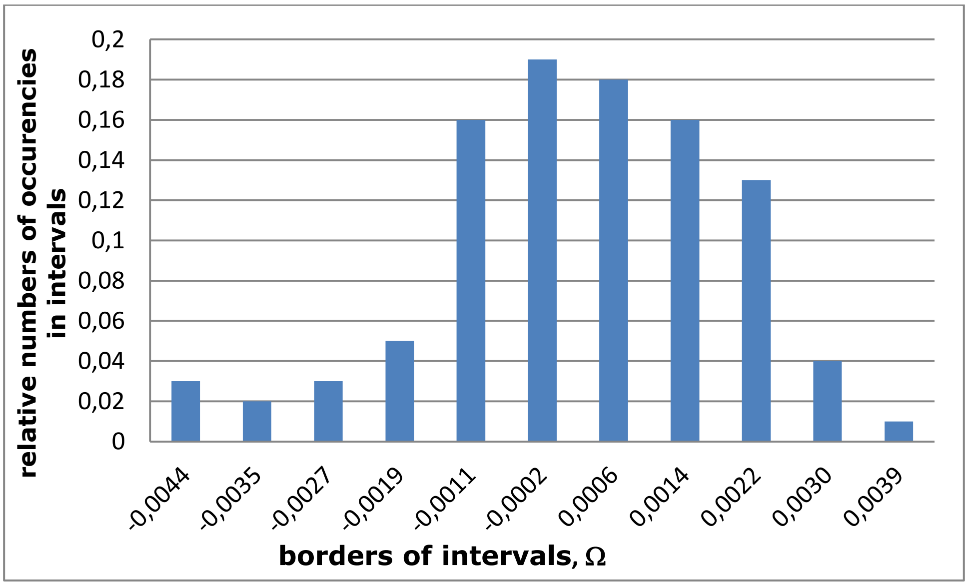


Fig. 9. Frequency of occurrences of apparent errors.

10. Convert borders of intervals (x axis) expressed in Ω, to relative values against standard deviation s_i, and vertical axis (y) as multipliers of frequency of error accuracies by width of each intervals (rys.10).

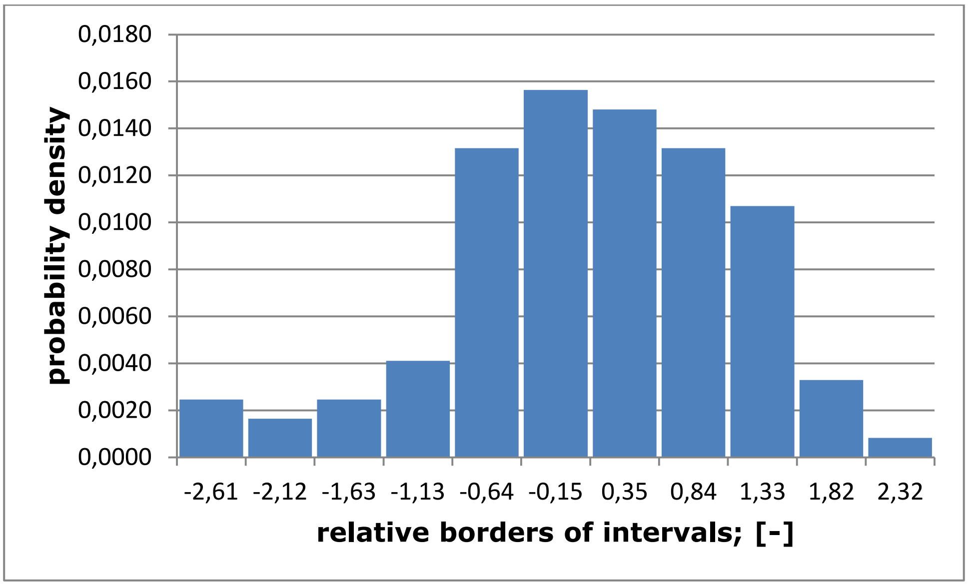


Fig. 10. Probability density relation vs. relative borders of occurrences

11. Such Probability density relation compare to the shape of probability density function of Normal distribution (rys.11).
    For comparison of experimental probability density function to Normal, the normalisation of area under experimental to the same are under normal distribution are required. Both areas should be equal to 1.

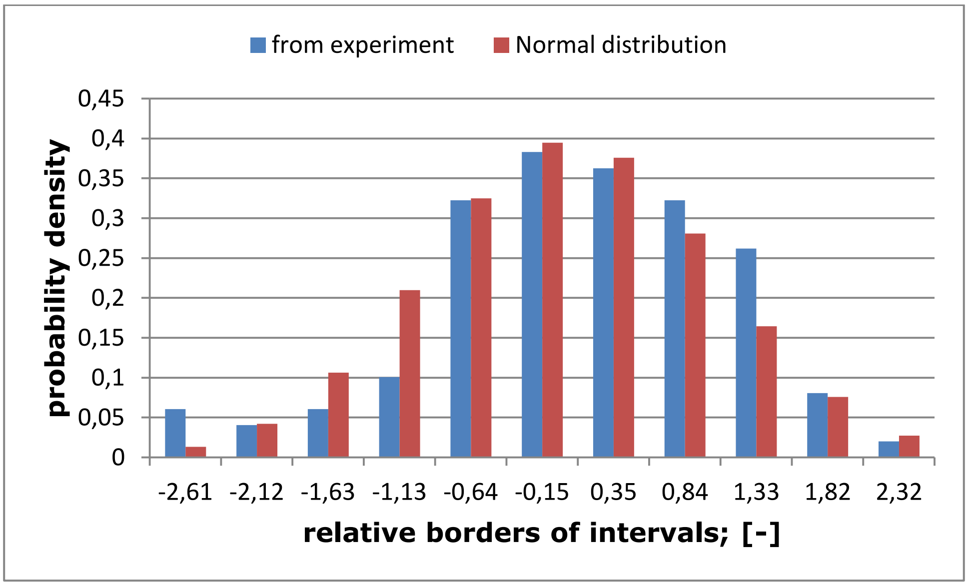


Fig. 10. Probability density of experimentally apparent errors (in blue) and probability density function of Normal distribution (in red).

12. Standard deviation of mean is a better estimation then standard uncertainty of individual observations. Std

$$s_{n(n-1)} = \sqrt{\frac{\sum_{i=1}^{n}(\Delta_i)^2}{n(n-1)}}$$

13. Calculation of uncertainty of type B from data specified by instrument producer: $u_{BRx} = \frac{1}{\sqrt{3}}\Delta_{grR} = \frac{1}{\sqrt{3}}\left(0{,}15\%R_{rdg} + 0{,}002\%R_n\right) = \frac{1}{\sqrt{3}}\left(0{,}15\%\overline{R_x} + 0{,}002\%R_n\right)$

14. Expanded uncertainty for coverage probability of 95 %, for p=0,95

    a. if u_B>>u_A *; that* $U_{Rx} \approx U_{BRx\ p=0{,}95} = 0{,}95\sqrt{3}u_{BRx}$ *U*

    b. *if* u_B<<u_A *; that:* $U_{Rx} \approx U_{ARx\ p=0{,}95} = 1{,}96 \cdot u_A = 1{,}96 \cdot s_{n(n-1)}$

    c. *otherwise:*

    $$u_{R_x} = \sqrt{u_{AR_x}^2 + u_{BR_x}^2}$$

    and in such case: $U_{R_x} \approx 2 \cdot u_{R_x} = 2 \cdot \sqrt{u_{AR_x}^2 + u_{BR_x}^2}$ for p=0,95

FINAL REMARKS:

ver. 6

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LABORATORY OF MEASUREMENTS

EXPERIMENT NO:	**1**
EXPERIMENT TITLE:	**Investigation of uncertainties in measurements of electrical quantities**

LABORATORY GROUP		***Program/Term***	
No.	***STUDENT'S NAME***		***ID***
1			
2			
3			
4			

Lecturer:	
Data date of experiment:	
Data of submitted report :	
Mark:	
Comments:	