TECHNICAL UNIVERSITY OF LODZ

INTERNATIONAL FACULTY OF ENGINEERING

MEASUREMENTS

LABORATORY EXERCISE # 3

Type A evaluation of standard uncertainties

1. Aim of the exercise

The laboratory exercise is aimed at recognising the method of evaluation of standard uncertainty by the statistical analysis of a series of observations on the basis of the measurement of time-constant temperature of hot wire and unsteady temperature of the heater.

2. Introduction

The assumed measurement method of a given physical quantity can become a source of the systematic error (mean result for a large number of repeated measurements of the same measurand[1] minus a true value of the measurand) of the obtained result. It can be followed from the method the measured quantity is sampled, measurement signal transformation, and from the fact that a significant influence of changes in one or several outer factors during the measurement time may result with a given method and they have not been taken into account, as well as from the assumed procedure of measurement result processing, etc.

A choice of the measurement method should be preceded by a detailed analysis of the measurement process that will allow for identification of possible sources of systematic errors. If, as a result of such an analysis, some quantities that vary but have not been considered in original and that exert an influence on the measurement result are found, then the simultaneous measurement of these quantities, which will be included in the calculation formulas that correct the measurement result, can become indispensable.

These issues are presented in this exercise on the example of the temperature measurement of the hot wire and unsteady temperature at the outlet of the heater. Measurements are conducted by means of thermocouples whose input signals are measured with a deflection method with analogue and digital instruments.

[1] Measurand – specific quantity subject to measurement

2.1. Temperature measurement of solid body surfaces

The methods for the temperature measurement of solid body surfaces can be divided into contact and contact-free ones. As contact transducers of temperature measurement most often are used thermocouples, rarely - thermistors (thermally sensitive resistors whose prime function is to exhibit a large, predictable and precise change in electrical resistance when subjected to a corresponding change in body temperature), and exceptionally - metal resistance thermometers. For an approximate determination of the surface temperature, thermometric paints that change their colour in a continuous or discontinuous way along with changes in temperature are used as well. Recently, liquid crystals or thermal imaging (infrared) cameras are often used in visualisation of temperature fields and heat patterns.

Thermocouples can be made of two wires characterised by various thermo-electrical properties or of thin layers of two different metals deposited on the surface of the body under investigation. They allow in practice for a point temperature measurement on the surface in a direct contact with the measuring junction of the thermocouple. Their important advantage is also an easiness of preparation and installation depending on measurement conditions. Miniature pearl and plate thermistors glued to the surface investigated are used in these measurements as well.

If the surface of a solid body is available for direct observation or through fibreglass, contact-free methods based on heat transfer via radiation can be applied. This radiation is especially intensive in high temperatures and then it is employed in temperature measurements with pyrometers.

Below, the temperature measurement by a contact method with thermocouples will be presented. This measurement is most often made with thermocouples soldered or welded to the surface under investigation. The heat transfer between the surface, whose temperature is measured, and the measuring junction of the thermocouple takes place via conductivity. In this method, three basic sources of systematic errors occur that follow from:

- Influence of the transducer on the temperature distribution of the body under investigation in the direct neighbourhood of the measuring point,
- Finite value of the heat transfer resistance between the thermocouple and the surface under investigation,
- Temperature distribution in a sensitive element of the measuring transducer.

While placing a temperature transducer in the measuring point, we disturb the existing heat equilibrium. Thermal conductivity, which causes a generation of a new state of equilibrium, occurs along the thermocouple wires. The temperature T that will establish in the measuring point in this changed state of heat equilibrium differs from the temperature in this point before the measuring transducer is placed in it.

Under actual conditions of the measurement, the temperature distribution along the wire length depends also on the heat transfer between the lateral surface of the wires and the surroundings. The discussed interaction is especially important in the temperature measurement of the body surface characterised by a low heat storage capacity.

A decrease in the values of systematic errors in the direct contact method of the time-constant temperature measurement is possible through:

- Decrease in the thermocouple wire diameter (a decrease in the thermal conductivity along the wires – a decrease in thermal conductivity is impossible in practice – the material the wires are made of is selected according to some other more vital issues),

- Minimisation of the heat resistance of the junction between the thermocouple and the body surface (the surface should be cleaned from oxides, grease and other impurities, application of special pastes with high heat conductivity),
- Miniaturisation of the sensitive element of the transducer (increase in the temperature distribution),
- Application of specially designed transducers with heat loss compensation.

Static characteristics of thermocouples, i.e. the dependency of the thermoelectric force generated (*STE*) as a function of the temperature difference of their junctions for typical thermocouples, are standardised in individual countries (e.g. in Poland PN-89/M-53854); they can also be found in literature. In Table 1 metals that are most often used in thermocouples and the *STE* corresponding to them, measured with respect to platinum (temperatures: hot junction 373 K, cold ends T_{zk} = 273 K) are presented.

Table 1

Kind of metal	STE [mV]
Copel (56 % Cu, 44 % Ni)	- 4.00
Constantan (55% Cu, 45% Ni)	- 3.51
Nickel-Aluminium (95% Ni, 2% Al)	- 1.90
Nickel	- 1.48
Platinum	0.00
Rhodium-Platinum (90% Pt, 10% Rh)	0.64
Iron	1.89
Nickel-Chromium (85% Ni, 12% Cr)	2.20
Chromel (90% Ni, 10% Cr)	2.81

The measurement of *STE*, which represents the measured temperature, can be made by a deflection or zero method. The schema of the measuring systems for both the methods are shown in Fig. 2.1.1.

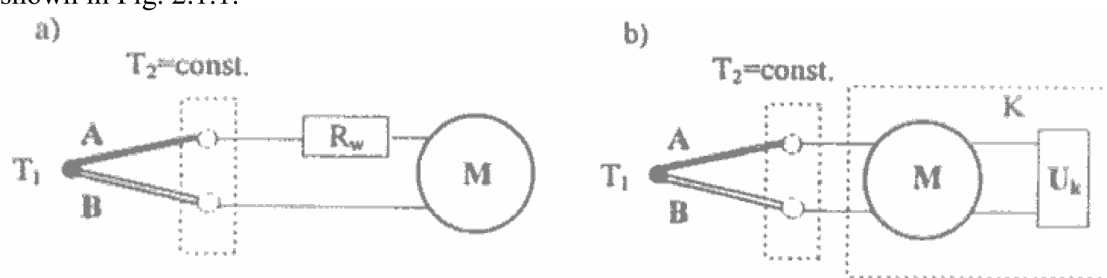

Fig.2.1.1. Schematic view of the measuring system of the thermoelectric force (STE) by:
a) deflection method; b) zero method
T_1 –temperature measured (hot junction), T_2 – ambient temperature (cold endings),
A, B – thermocouple wires, M – measuring device,
R_w – compensating resistor, U_k – STE compensating supply

The thermocouple is a generative transducer in which, owing to the temperature difference T_1 - T_2, the thermoelectric force *STE* that causes a passage of the current *I* in the circuit is generated. The value of this current (for the deflection method) is described by the relation:

$$I = \frac{STE}{R_t + R_p + R_w + R_m} = \frac{STE}{R_z + R_m} \tag{2.1.1}$$

where: R_t, R_p, R_w, $R_{m,}$ – resistance of the thermocouple wires, connecting wires, compensating resistor and meter, respectively.

The resistance $R_z = R_t, + R_p + R_w$ is so called the resistance of the outer circuit. The meter *M* measures the voltage *U*

$$U = I \cdot R_m = STE \cdot \frac{R_m}{R_L + R_P + R_W + R_M} \tag{2.1.2}$$

whose value differs from the thermocouple output signal, i.e. from *STE.* This value reflects the systematic error of the method under discussion. In order to eliminate it, meter manufacturers require that a defined, constant resistance of the outer circuit R_z should be maintained (for this purpose, a resistor R_w with adjustable resistance is supplied with the meter, so that the sum of resistances $R_t + R_p + R_w$ can be established to achieve the required value) and they mark a graduation on the meter scale that accounts for the required correction eliminating the systematic errors that results from the nominal value of R_z. In the case when during the measurement the true value of the outer circuit resistance R_z' is different from its nominal value R_z, the measurement result *U*' is burdened with the systematic error ΔU

$$\Delta U = U - U' \frac{\Delta R_z}{R_z + R_m} \tag{2.1.3}$$

where: $\Delta R_z = R_z' - R_z$ – change in the outer circuit resistance.

From relation (2.1.4) it follows that the value of the error ΔU:

- is a linear function of the resistance change ΔR_z,
- decreases as the meter resistance R_m increases.

When there is a linear relation between the voltage *U* and the measured temperature *T*, relation (2.1.3) makes it possible to introduce directly the correction ΔT eliminating an influence of this systematic error. In practice changes in resistance can result from a change in wires conductivity due to their corrosion, changes in the ambient temperature, and, first of all, changes in transfer resistance on various kinds of connections (e.g. a clamp). These changes are especially dangerous for systems with a meter characterised by a low inner resistance (characteristic feature of analogue temperature meters).

In the compensating method (Fig. 2.1.1.b) the measured *STE* is compared with the oppositely supplied compensating voltage U_k. In the equilibrium state, when $STE = U_k$, there is no passage of current ($I=0$) in the measuring system. A lack of the current passage makes the zero compensating method independent of an influence of changes in the outer circuit resistance on the measurement result.

3. Description of the measurement installation

The measurement installation is composed of two independent parts in which the measurement of time-constant temperature of the hot wire and the measurement of an unsteady flow are carried out.

3.1. Installation for the temperature measurement of the body with low heat storage capacity

The scheme of the measurement installation is presented in Fig. 3.1.1. The object under investigation is nickel-chrome wire (1) through which electric current passes and heats it up. The wire is spread between two brackets (2).

The temperature of the wire surface is measured with two thermocouples (3) fixed to its surface. Both thermocouples are made of Ni-Cr vs. Ni-Al (type K) wires and differ only as far as their diameter is concerned (ϕ 0.1 and ϕ 0.3 mm). Cold endings of the thermocouples are placed in thermostat (4) made in the form of a metal block whose temperature is equal to the ambient temperature. In the circuit of the thicker thermocouple O_2, two disturbing resistors (7) are placed, creating thus two additional circuits $O_{2'}$ and $O_{2''}$. The measurements of *STE* are carried out by a deflection method with analogue (8) and digital (9) meter (multirange meter). A choice of the proper measuring system and the meter is done in the panel of switches (6). The two first switches allow for a selection of the measurement method (*W* – deflection method, *Z* – zero method), and in the case of the deflection method, they allow for a selection of the kind of the measuring instrument (*A* - analogue, *C* – digital) as well. The others switches allow for a selection of one out of four measuring circuits (O_1, O_2, $O_{2'}$, $O_{2''}$).

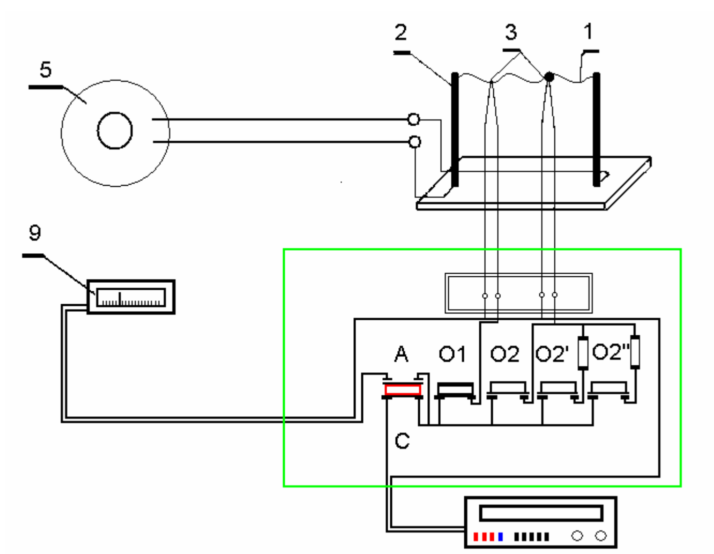

Fig. 3.1.1. Scheme of the measurement installation for the temperature measurement of the element characterised by low heat storage capacity
1 – element under investigation, 2 - brackets, 3 – thermocouple, 4 – thermostat, 4 – power supply cable, 6 - panel of switches, 7 - disturbing resistors, 8 – analogue meter, 9 – digital meter, 10 – compensator

Meter (8) is graduated in temperature units, its resistance R_m and the nominal resistance R_z of the outer circuit are given on the front panel. The resistance of the circuits O_1 and O_2 is close to the nominal resistance R_z. Digital meter (9) is graduated in voltage units. In order to determine the temperature measured, we should know the static characteristics of the thermocouple used.

3.2 Installation for the temperature measurement of the unsteady flow

The object under investigation is a heater (one should be careful not to touch the outlet parts of the heater). The temperature measurement is made with thermocouple of the type E. The deflection method with a digital voltmeter has been chosen to measure temperature [^{o}C]. The temperature of the heater outlet flow is determined from the calibration characteristics of the thermocouple $T = f(STE)$.

4. Description of the exercise

During the exercise, the time-constant temperature of the wire surface (the element with low heat storage capacity) and of the flow (the flow with high heat storage capacity) should be measured, the obtained results should be processed and the systematic errors related to the assumed method of measurement should be found. Then the results should be corrected and the uncertainties of the measurement resulting from random errors are to be determined as well.

4.1. Temperature measurement of the element with low heat-storage capacity

Having got acquainted with the measurement installation, its individual elements, the description of the planned measurements and having got consent of the instructor, one should:

- determine the nominal resistance R_m of meters (8) and (9),
- switch digital meter (9) and compensator (10) on,
- switch digital multimeter (9) to the measurement of resistance and measure the resistance of the circuits: O_1, O_2, $O_{2'}$, $O_{2''}$, record the results in Table 1 of the report,
- switch the supply of the resistance wire on and, according to the instructions, adjust the value of the supply voltage.

When the heat conditions of the system become steady (for a given supply voltage of the resistance wire (1)), the measurements can be started (the results are to be recorded in Table 1 of the report), according to the following scheme:

a) measurement of the wire temperature by a deflection method with digital meter (9):

- measurements are to be conducted as above when the second of the first two switches is fixed in "*C*" (*digital*) position, the first one should rest in the same position "*W*",

next, after having switched on subsequently the circuits O_1, O_2, $O_{2'}$, $O_{2''}$, the measured wire temperatures (as *STE* in mV) from the digital meter (9) can be read respectively,

b) measurement of the wire temperature by a deflection method with analogue meter (8):

- the first two switches should be fixed in the positions "*W*" (*deflection*) and "*A*" (*analogue*),
- next, after having switched on subsequently the circuits O_1, O_2, $O_{2'}$, $O_{2''}$, the measured

wire temperatures (directly in ^{0}C) from the analogue meter (8) can be read respectively,

c) next the supply voltage of the resistance wire (1) should be changed to another value indicated by the instructor and after having established a new heat equilibrium conditions, the measurements (points a - b) are to be repeated. The same method should be used all voltage values (at least for seven different wire temperatures).

After having finished the measurements, the supply voltage and the measuring devices should be turned off, and then the students should start processing the results:

a) the measured values are to be transformed into the temperatures t_i [K] measured with respect to the reference temperature,

b) the corrections Δt_i that eliminate systematic errors resulting from a difference between the true value of the outer circuit resistance R_{zi} and its nominal value R_z for individual circuits for each circuit are to be calculated:

$$\Delta t_i = \frac{R_{zi} - R_z}{R_m + R_z} \cdot t_i \tag{4.1.1}$$

c) the corrected values of the measured temperatures are to be calculated:

$$T_i = t_i + \Delta t_i + t_{rel} + 273.15 \quad [\,\mathrm{K}\,] \tag{4.1.2}$$

d) the relations of the wire temperature measured by a deflection method with analogue meter in function of the temperature measured by a deflection method with digital meter are to be estimated with a linear equations and the plots $T_a = f(T_d)$ (for all the circuits O_1, O_2, $O_{2'}$, $O_{2''}$ on the same figure) are to be drawn (Fig. 4.1.1) and linearization coefficients are to be calculated.

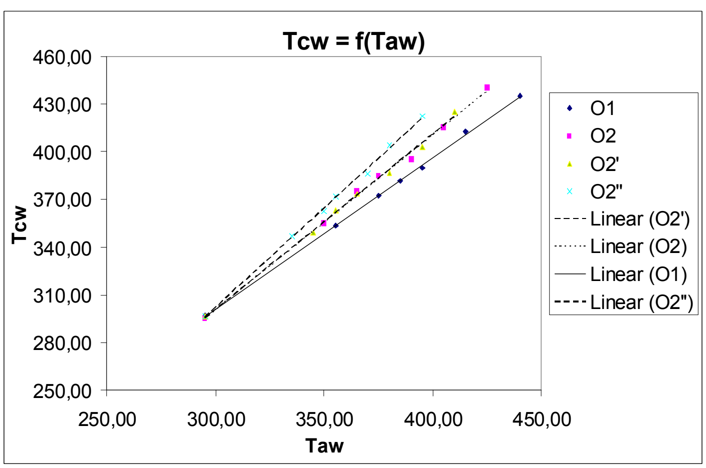

Fig. 4.1.1. Temperature correlation (example)

4.2. Temperature measurement of the heater flow

Before getting started the measurements, one should get acquainted with the meteorological properties, design and operation of the multirange digital voltmeter, and one should select a proper measurement range. At least 60 temperature measurements of the outlet flow at different time are to be performed using a Type K thermocouple. The results are to be recorded in Table 2 of the report.

The obtained results should be processed in the following way:

a) the measured values should be recalculated into the values of the temperature T_I [K],

b) a point estimation of the obtained random sample of n measurements is to be carried out:

- the mean value of the measured temperature

$$\bar{T} = \frac{1}{n}\sum_{i=1}^{n} T_i = ...[K] \tag{4.2.1}$$

- the empirical standard deviation

$$S_T = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(T_i - \bar{T})^2} = ...[K] \tag{4.2.2}$$

  as the measure of the result scattering in the obtained sample,
- the scattering range R of the obtained results from the sample:

  $T \in < T_{min} =$ .... [K]; $T_{max} =$ .... [K] > (4.2.3)

c) the obtained results are to be classified into 6 main bins:

- the relative frequency is a total count of the number of m_i results values that fall within the bin divided by the whole number of measurements (m_i/n),
- the probability density *f(u)* and observed distribution in the bins should be calculated,
- the calculated densities (Fig. 4.2.1) in the form of a bar chart (histogram) as the distribution of the probability density function *f(u)* and a comparison with the normal standardised distribution should be presented.

Histograms display data as a series of groups, called bins. Each bin represents a range of values on the abscissa axis. The height of a bin represents the probability density function that is characteristic for that bin range or a number of results that fall into that bin range. One can manually set the number of bins and the bin size, or determine a bin range automatically as S_T.

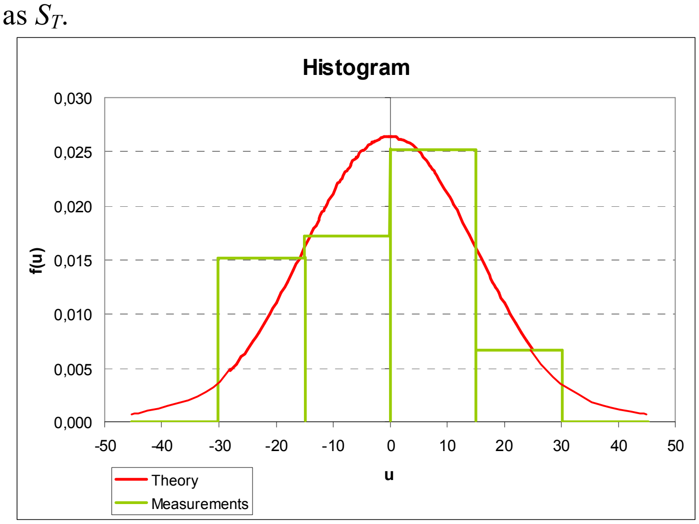

Fig. 4.2.1. Example of the probability density function and histogram

d) the probability density function for the whole population:

$$f(T) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(T-\mu)^2}{2\sigma^2}} \tag{4.2.4}$$

assuming that the estimator of the expected value μ is the mean value $\bar{T}$ of the sample, and the estimator of the standard deviation σ is the empirical standard deviation S_T of the sample.

e) the empirical standard deviation of the mean value $S_{\bar{T}}$ and the confidence interval for the values of the mean temperature (the degrees of freedom = n-1) are to be calculated,

f) the final measurement result of the mean temperature of the hot plate should be recorded:

$T = \overline{T} \pm \Delta\overline{T}$ [K] with the probability (*make your own choice*) %.

5. Final remarks

The report should include:

- aim of the exercise,
- short description of the activities carried out,
- measurement tables,
- description and comparison of the accuracy of the measurements with individual meters,
- description and explanation of the influence of the thermocouple wire diameter on the measurement of time-constant temperature of the surface of the element characterised by low and high heat storage capacity,
- bar chart of the probability density function for the conducted measurements of the plate surface,
- conclusions and remarks concerning the exercise.

Check questions:

- *design and principle of operation of the thermoelectric temperature transducer,*
- *kinds of thermocouples being used in temperature measurements and the range of their application,*
- *classification of measuring methods, their advantages and disadvantages,*
- *methods of measurement of the input signal from the thermocouple,*
- *what is the difference between a raw and corrected measurement result?*
- *systematic error, random uncertainty – sources of occurrence, identification methods, possibilities of elimination,*
- *explain the influence of a measuring instrument on the phenomenon source on the basis of some selected examples,*
- *parameters of the normal distribution of the random variable,*
- *aim and way of the statistical processing of measurement results.*

Appendix 1: **Student's t test -** Values of '**t**'

Note: This table does not show all degrees of freedom. If you want a value between, say 30 and 40, then use the value for 30 DoF.

Degrees of Freedom f	**α = 0.1**	**α = 0.05**	**α = 0.02**	**α = 0.01**	**α = 0.001**
1	6.314	12.706	31.821	63.657	636.619
2	2.920	4.303	6.965	9.925	31.598
3	2.353	3.182	4.541	5.841	12.941
4	2.132	2.766	3.747	4.604	8.610
5	2.015	2.571	3.365	4.032	6.859
6	1.943	2.447	3.143	3.707	5.959
7	1.895	2.365	2.998	3.499	5.405
8	1.860	2.306	2.896	3.355	5.041
9	1.833	2.262	2.821	3.250	4.781
10	1.812	2.228	2.764	3.169	4.587
11	1.796	2.201	2.718	3.106	4.437
12	1.782	2.179	2.681	3.055	4.318
13	1.771	2.160	2.650	3.012	4.221
14	1.761	2.145	2.624	2.977	4.140
15	1.753	2.131	2.602	2.947	4.073
16	1.746	2.120	2.583	2.921	4.015
17	1.740	2.110	2.567	2.898	3.965
18	1.734	2.101	2.552	2.878	3.922
19	1.729	2.093	2.539	2.861	3.883
20	1.725	2.086	2.528	2.845	3.850
21	1.721	2.080	2.518	2.831	3.819
22	1.717	2.074	2.508	2.819	3.792
23	1.714	2.069	2.500	2.807	3.767
24	1.711	2.064	2.492	2.797	3.745
25	1.708	2.060	2.485	2.787	3.725
26	1.706	2.056	2.497	2.779	3.707
27	1.703	2.052	2.473	2.771	3.690
28	1.701	2.048	2.467	2.763	3.674
29	1.699	2.045	2.462	2.756	3.659
30	1.697	2.042	2.457	2.750	3.646
40	1.684	2.021	2.423	2.704	3.551
60	1.671	2,000	2.390	2.660	3.460
120	1.658	1.980	2.358	2.617	3.373
∞	1.645	1.960	2.326	2.576	3.291