TECHNICAL UNIVERSITY OF LODZ

INTERNATIONAL FACULTY OF ENGINEERING

MEASUREMENTS

LABORATORY EXERCISE # 2

Type B evaluation of standard uncertainty

1. Aim of the exercise

The laboratory exercise is aimed at recognising the method of evaluation of standard uncertainty by means other than the statistical analysis of a series of observations on the basis of the measurement of pressure with a hydrostatic pressure gauge (manometer).

2. Introduction

Every measurement process carried out under real conditions is subject to an influence of outer and inner factors. As a result of this influence, the measurement result x_i differs from the true value $\dot{x}$ of the measured quantity X. This difference

$$\dot{\Delta} x_i = x_i - \dot{x} \tag{2.1}$$

is referred to an absolute true error.

The occurrence of this error is affected by random and systematic phenomena and by mistakes. The component of the error $\dot{\Delta} x_i$ that results from the influence of unidentified (random) factors, i.e. such factors that are not described by means of explicit mathematical models, is called a random error $\dot{\Delta} x_{ri}$. The second component of the error $\dot{\Delta} x_i$ results from the occurrence of systematic phenomena during the measurement and, owing to this, it is possible to identify them through an analysis of the measurement process and to correct the results of the measurement. This component of the true error $\dot{\Delta} x_i$ is called a systematic error $\dot{\Delta} x_{si}$.

The random error $\dot{\Delta} x_{ri}$ is a difference between the measurement result and an average of the infinite number of measurements of the same measured quantity that are carried out under repeatability conditions, whereas the systematic error $\dot{\Delta} x_{si}$ is defined as a difference between an average of the infinite number of measurements of the same measured quantity that are made under the repeatability conditions and the true value of the measured quantity. Assuming that the measurement result is not burdened with any error, we have:

$$\dot{\Delta} x_i = \dot{\Delta} x_{si} + \dot{\Delta} x_{ri} \tag{2.2}$$

The values of the systematic error and the random error, as well as the causes why they occur are unknown, just like the true value of the measured quantity.

Systematic influences on the measurement result should be accounted for as one of the first activities while processing the results of the measurement process. Identification of the systematic error consists in both determination of its value and in finding the circumstances, place or causes of its occurrence. Systematic errors can be especially dangerous when they occur in the measurement result, and the person who conducts the measurement does not even suspect their occurrence. In order to identify the systematic error, the applied measurement method, apparatus, way the measurements are made, effects of outer and inner factors on the measurement results should be examined in details.

Below some exemplary sources of the occurrence of the systematic error in the pressure measurement with a hydrostatic pressure gauge will be presented.

A hydrostatic pressure gauge is a manometer in which the measured pressure is transformed into the corresponding manometer liquid column height. The measurement of this height that reflects the measured pressure can be carried out by means of a deflection method and a zero method. We distinguish single-, double- and multi-tube (or battery of tubes) manometers with tubes that are vertical or positioned at some angle with respect to the horizon (Fig. 2.1). The liquid level in the tube is measured with respect to the bottom meniscus for those liquids that form a concave meniscus and with respect to the upper meniscus for the liquids that form a convex meniscus (Fig. 2.1.e).

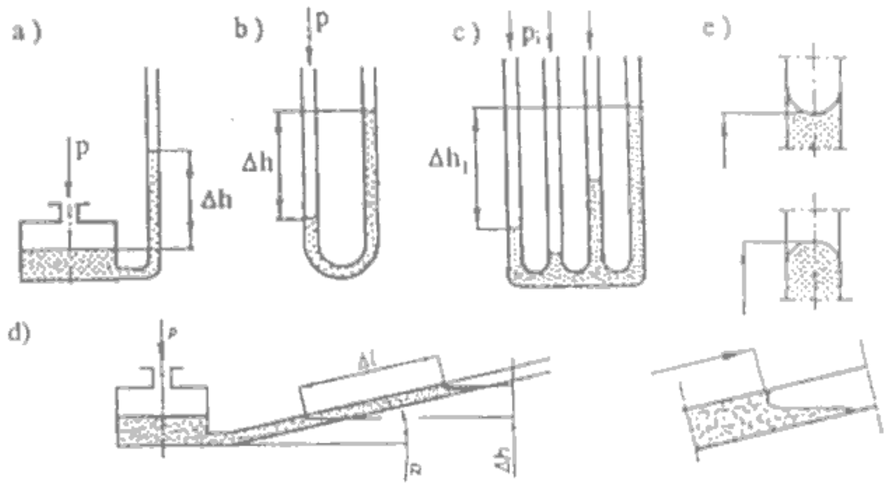

Fig.2.1. Hydrostatic pressure gauge: a) single – tube, vertical; b) double – tube, vertical (U-tube); c) multi-tube, vertical; d) single-tube, slanting (micromanometer); e) the way the liquid column height is measured

The value of the measured pressure is determined from the formula:

$$p = \frac{\Delta l \cdot \rho_{cm} \cdot g}{i_m} \tag{2.3}$$

where: ρ_{cm} – manometer liquid density,
g – acceleration of gravity,
i_m – micromanometer ratio (for the vertical manometer $i_m = 1$).

The manometer ratio i_m determines the ratio of the displacement Δl of the liquid column along the tube length to the displacement Δh measured in the vertical plane (see Fig. 2.1.d).

$$i_m = \frac{\Delta l}{\Delta h} = \frac{1}{\sin\alpha} \tag{2.4}$$

For a given manometer, the expression $k = \rho_{cm}g/i_m$ has a constant value, thus the conversion equation of the manometer assumes the form of the linear relation

$$p = k\,\Delta l \tag{2.5}$$

where k is the sensitivity of this manometer.

Systematic errors in hydrostatic pressure gauges can be caused by:

- change in the manometer liquid density,
- changes in the ambient temperature (a complex effect – it affects the manometer scale length, liquid density, surface tension, etc.),
- effect of capillarity,
- inaccuracy in setting the zero level (for $p = 0$) and the angle, at which the manometer tubes are positioned,
- curvature of the manometer tubes,
- others – related directly to given, current conditions of the measurement.

A difference between the value of the manometer liquid density ρ_{cm} assumed in equation (2.3) and its true value (during the measurement) $\dot{\rho}_{cm}$ causes an occurrence of a multiplicative systematic error (proportional to the pressure value measured) – $(\Delta p_{si})_\rho$ (Fig. 2.3.a).

$$(\Delta p)_p = \frac{\rho_{cm} - \dot{\rho}_{cm}}{\rho_{cm}} \cdot p_1 \tag{2.6}$$

Water, mercury, alcohol, toluene, carbon tetrachloride are most often used as manometer liquids. The density of a manometer liquid is a function of its temperature.

Changes in density of selected manometer liquids as a function of their temperature are presented in Table A1 (Annex 1). It is necessary to take into account these changes during accurate measurements of pressure.

The phenomenon of capillarity, which results from the action of surface tension forces, causes that the liquid level in the tube rises or lowers (Fig. 2.2). The value of a change in the liquid level in the tube can be found from the relation:

$$\Delta h - \frac{4\sigma}{\rho g d}\cos\vartheta \tag{2.7}$$

where: σ - surface tension [N/m],
d - tube diameter [m],
ρ - liquid density [kg/m^3],
ϑ - angle between the tangent to the liquid surface and the generating line of the tube,
g - acceleration of gravity [m/s^2].

The value of the angle ∂ depends not only on the kind and material of the surface moistened, but also on the degree of its pollution.

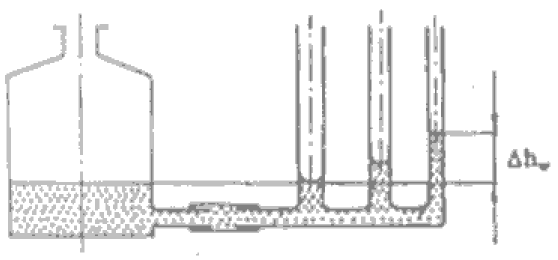

Fig. 2.2. Change in the liquid level in capillary tubes

For vertical (glass) manometers, a change in the liquid level caused by capillarity is determined from the empirical relationship

$$\Delta h_w = \frac{A}{d}[mm] \tag{2.8}$$

The value of the empirical constant A is, respectively: 300 mm^2 for water, -10 mm^2 for mercury, 10 mm^2 for spirit, and 13 mm^2 for toluene in contact with air.

Capillarity introduces an additive systematic error (Fig. 2.3.b).

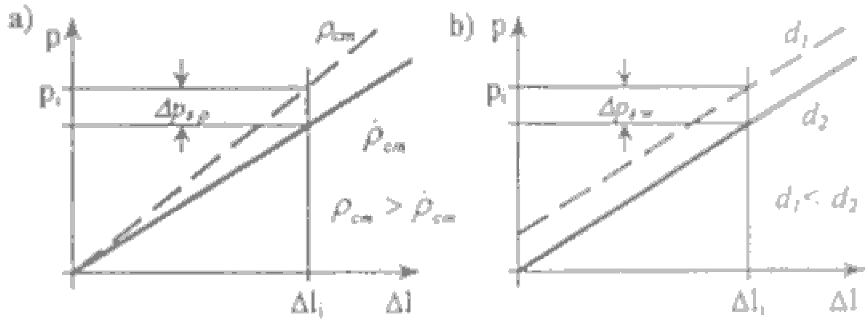

Fig. 2.3 Systematic errors in the hydrostatic pressure gauge:
a) caused by a change in the liquid density, b) caused by capillarity

An application of a vessel with the cross-section A_n that is much bigger than the tube cross-section field A_r as one of the manometer tubes (Fig. 2.1.a, 2.1.d) simplifies the pressure measurement – if $A_n/A_r >>1$ is high enough, it can be assumed that the liquid level in the vessel is constant and does not change in practice under the influence of the supplied pressure p. Comparing the changes in the liquid volume in the vessel and in the tube, we can write

$$A_n \cdot \Delta l_n = A_r \cdot \Delta l_r \qquad \Delta l_n = A_r/A_n\, \Delta l_r \tag{2.9}$$

when $A_r/A_n << 1 \Rightarrow \quad \Delta l_n << \Delta l_r \quad \Rightarrow \quad \Delta l_n \approx 0$

where – Δl_n, Δl_r - change in the liquid level in the vessel and in the tube, respectively.

The pressure is thus measured as a change in the liquid level in the manometer tube.

If a change in the liquid level is neglected, a systematic error equal to the value directly proportional to the measured pressure (the measured pressure is lower that its true value) occurs:

$$(\Delta p_{si})_{An} = p_i \cdot \frac{A_r}{A_n} \tag{2.10}$$

The systematic error of the measurement result can be eliminated by:

- introduction of corrections to the raw measurement result (with a value opposite to the value of the error identified),
- elimination or limitation of the error sources (e.g. in the example described here, the tubes with the same, possible large diameter, in which capillarity exerts a negligible small effect on the value of the measured pressure, are used in the manometer),
- compensation for the errors (an introduction of active elements in the instrument whose action is opposite to the phenomena being the sources of systematic errors).

A selection of the method for systematic error elimination depends on the kind of the quantity measured, systematic errors that occur, instrument design, method for processing the results, etc. After the compensating correction of the systematic error is introduced, the measurement result is called the corrected result.

Assuming that as a result of the detailed analysis of the measuring process, all the quantities whose change causes an occurrence of the systematic error have been found and that these quantities do not change during the measurement, we can say that the systematic error is reproducible and thus it can be determined through calibration.

3. Description of the installation

The measuring installation (Fig. 3.1) is adapted for determination of the systematic error values of the hydrostatic manometer through its calibration by means of a comparative method. The installation consists of three main parts:

- manometer under investigation (1),
- higher standard manometer (2),
- pressure generator (3).

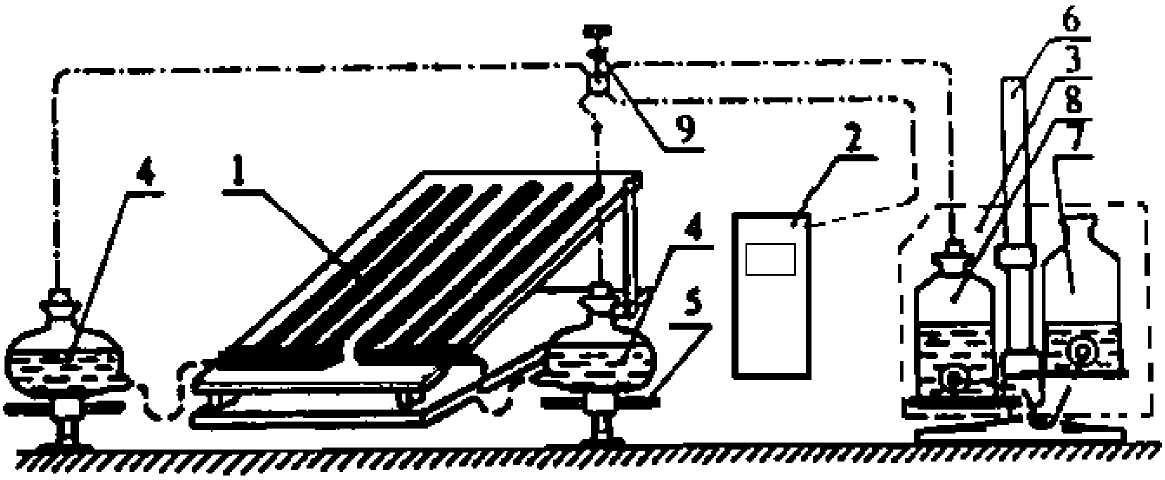

Fig. 3.1. Schematic view of the measuring installation
1 – manometer under investigation, 2 – standard manometer, 3 – pressure generator, 4 – compensating vessels, 5 – compensating holder, 6 – stand, 7 - top vessel, 8 – bottom vessel, 9 – valve

The manometer under investigation is a battery inclined manometer with a variable ratio (i_m = 1, 2, 5, 10, 20). It contains two sections (batteries) of tubes mounted on a metal plate and connected with compensating vessel (4). The plate with manometer tubes, hinged-fixed to the base, can be positioned at different angles with respect to the horizontal plane. A displacement of compensating vessel (4) with compensating holder (5) allows for setting the liquid level in a given section of the tubes.

The measured pressure, supplied from generator (3) to compensating vessels (4), causes a displacement of the liquid in the manometer tubes. A level of the liquid in a given tube indicates the pressure measured by it – we neglect a change in the liquid level in vessel (4). The manometer is made of tubes with various inner diameters (e.g. 6, 4, 1.5 mm) that cause an occurrence of systematic errors of different values.

Pressure generator (3) is made as two connected vessels (7) and (8) mounted on the stand. The surface of the liquid in vessel (7) is connected with the surroundings (ambient pressure), whereas in vessel (8) - with a closed, tight chamber (connecting pipes, compensating vessels

(4), closed space in standard manometer (2)). When vessels (7) and (8) are positioned at various height, the liquid flows from the top vessel to the bottom one as long as the air pressure in the closed volume connected to vessel (8) does not balance the hydrostatic pressure resulting from the difference in the liquid level in both the vessels.

Note: The valve (9) that connects the closed chamber with the surroundings can be opened only when the liquid levels in vessels (7) and (8) are approximately equal.

The higher standard electronic manometer measures directly the gas pressure. The pressure measured is supplied via connecting pipes to closed vessels. The vessels are partially filled with the liquid and connected by means of elastic pipe.

4. Description of the exercise

The exercise consists of two parts:

- tests of the hydrostatic manometer,
- pressure measurement with the calibrated manometer.

4.1. Tests of the manometer

The tests of the manometer are carried out in the following way (denotations according to Fig. 3.1):

a) the density of the manometer liquid ρ_m in tubes 1, 2 and 3 should be determined basing on the table,
b) after having opened the valve (9), one should level and set to zero the standard manometer,
c) the manometer under investigation ought to be levelled and the liquid level in the tubes should be observed, the sources of various liquid levels in individual tubes have to be identified,
d) the liquid levels in tubes with the highest inner diameters (i.e. tubes 1 and 4) of the manometer under investigation should be set to zero (according to the bottom meniscus!), one should read the liquid levels in the remaining tubes and record them in Table 1 of the report, then the valve (9) should be closed,
e) by moving the vessel (7) of the generator (3), one should change the value of pressure p; when the pressure becomes steady, its value p_w and the levels of the liquid l_i in individual tubes of manometer (1) ought to be read with higher standard (electronic) manometer (2), next they have to be recorded in Table 1,
f) the above activities should be repeated to obtain a proper number of measurement points distributed uniformly along the manometer scale length (at least 10 measurements!),
g) on the basis of the measurements conducted, one should determine a relationship between the real pressure and that one indicated by individual tubes: $p_w = f(p_i)$ [Pa]; this relation should be described with a linear equation, whose constants are determined with the least square method, and, both results and function should be presented as graphics,
h) the actual ratio of the manometer i_r is to be determined from tube equations basing on directly measured pressures and equation. 2.3, and values of pressure p_i in every individual tubes are to be corrected and then the corrected values of pressure p'_i are to be recorded in Table 1,
i) the differences between the actual and measured values of pressure in individual tubes (estimated total measurement errors) are to be calculated

$$\Delta p_i = p_w - p'_i \qquad (4.1.1)$$

j) the above calculated values are to be recorded in Table 2 and the plots $\Delta p_i = f(p'_i)$ are to be drawn,
k) the relations $\Delta p_i = f(p'_i)$ are to be estimated with linear equations,
l) one should assume that the determined equations defines the changes in the estimated systematic error Δp_{si} of the measured pressure in individual tubes of the manometer under investigation, i.e. that they represent the changes in the difference of the average value of the infinite number of pressure measurements made under the repeatability conditions and the correct value (p_w) as a function of the measured pressure,
m) the equations describing the values of the estimated systematic errors $\Delta p_i = f(p'_i)$ for individual tubes and the values of estimated errors calculated from them Δp_{si} are to be written in the report (in Table 2). The sources of systematic errors are to be identified,
n) the random uncertainties $\Delta p_{ri} = \Delta p_i - \Delta p_{si}$ for individual tubes are to be calculated and plots of the random uncertainties $\Delta p_{ti} = f(p'_i)$ are to be drawn,
o) the values of the maximum random uncertainties $(\Delta p_{ri})_{\max}$ and the classes of the calibrated manometer (for every tube) are to calculated:

$$kl = \frac{(\Delta p_{rl})_{\max}}{z} \qquad (4.1.2)$$

As a result of the manometer calibration the following is to be obtained:

- value of the true ratio of the manometer,
- value of the liquid density in tubes 4, 5, and 6,
- values of corrections for every manometer tube,
- class of the calibrated manometer.

4.2. Measurement of pressure with the calibrated manometer

One, two or three different values of pressure with the calibrated manometer are to be measured by reading the changes of the liquid column position for each manometer tube - the results are to be recorded in Table 3 of the report.

The corrected values of the pressure measured with each manometer tube, taking into account the actual ratio of each manometer, are to be measured and the corrections $\Delta p'_i = -\Delta p_{si}$ that eliminate the systematic error Δp_{si} for every tube are to be determined.

The obtained results are to be compared and the average value of the measured pressure is to be calculated.

5. Final remarks

The report should contain:

- aim of the exercise,
- short description of the activities carried out,
- measurement tables,
- diagrams of pressure for individual tubes $p_w= f(p'_i)$ with the fitted lines determining the tendency,
- diagrams of total pressure errors for individual tubes $\Delta p_i= f(p'_i)$ with the lines determining the changes in systematic errors,
- diagrams of random pressure errors for individual tubes $\Delta p_{pi}=f(p'_i)$,
- identified sources of systematic errors appearing in the manometer under investigation, with the reason of their occurrence (for each tube),
- conclusions and remarks concerning the exercise.

Check questions:

- *sources of the divergence between the measurement result and the true value of the physical quantity measured,*
- *definition of the measurement error, systematic error, random error, measurement uncertainty, correction that eliminates the systematic error,*
- *what is the difference between the raw and corrected measurement result?*
- *discuss systematic errors: constant, variable (progressive, periodical, variable according to a certain principle) on the basis of some selected examples,*
- *what is the practical aim of the systematic error identification?*
- *what is the systematic error resulting from capillarity in the pressure measurement with a vertical manometer (i_m =1.0) made of glass tubes with the diameter 1.5, 4, 6 mm and filled with a) water, b) methyl spirit?*
- *what is the systematic error that follows from a change in the manometer liquid density: a) water, b) methyl spirit, resulting from a change of temperature from 20 ℃ to 30 ℃?*
- *what is the value of the systematic error resulting from the fact that a change in the liquid level in vessels (4) is not accounted for during calibration of the hydrostatic manometer? Is this error eliminated by calibration? Give reasons for your answer.*

Appendix
Table A1

Density of manometer liquids [kg/m^3] vs. temperature

Temperature [^{o}C]	Water	Mercury	Methyl alcohol	Toluene	Carbon tetrachloride	Tribromoethane
0	999. 8	13 995	810			
10	999.7	13 571	801	875	1 605	2 640
15	999.1	13 558				
20	998.2	13 546	792	864	1 588	2 617
25	997.0	13 534				
30	995.6	13 522	783	858	1 576	2 600
40	992.3	13 497	774	849	1 558	2 580
50	988	13 473	765	841	1 539	2 560
60		13 448	756			